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THE MATHEMATICS TEACHER

Volume XXXII Number 3

Edited by William David Reeve

The Professional Preparation of Mathematics Teachers

By F. L. Wren
George Peabody College for Teachers, Nashville, Tennessee

What is a profession? What is meant by professional preparation? A profession is a calling or occupation in which one has acquired significant knowledge and information which he uses in the instruction, guidance, and service of his fellowman. There are two equally important aspects of any true profession, namely, significant knowledge and effective technique. One cannot be efficiently professional if there is any distinct weakness in either aspect. A truly functional program of professional preparation must therefore place emphasis on the acquiring of knowledge significant to a chosen profession and also on the acquaintance with and use of the more efficient techniques of that profession.

There is a trichotomy of knowledge significant to the teacher of mathematics which might be classified under the headings of general knowledge, professional knowledge, and specialized knowledge. We are living in an age in which events take place very rapidly. This rapidity of development and its implications for future change not only tend to stagger the imagination but also to encourage a satisfaction in superficiality of information. Things happen too fast for any individual to be able to attempt a very thorough and systematic acquaintance with the fundamentals of all lines of development. We have to be satisfied with a type of superficial information along some lines. It is for such reasons as this that the teacher of mathematics must have a broad general education out of which he should be able to construct a cultural atmosphere in which he may stabilize his thinking and orient his appreciation of values. Such an informational background should be related to "the major areas of human experience" and designed to build up a more intelligent understanding of the part mathematics has played in the evolution of modern civilization and a deeper appreciation of its relation to social progress. Such "a program of general education for prospective teachers should acquaint them with the various institutions and forces that influence modern life and with the contributions that the major fields of learning have made and are making today to the progress of civilization."[1]

A professional attitude should be a *sine qua non* characteristic of every teacher of mathematics. By "professional attitude" is here meant an enthusiastic interest in mathematics as a chosen field of study and

[1] E. W. Knight. "Discussion: Directors of Needed Improvement in Subject Matter," *Yearbook Number XXVI of the National Society of College Teachers of Education* (Chicago: The University of Chicago Press, 1938), p. 16.

service, an inspired concept of the value of mathematics in the structure of civilization, and an eager readiness to interpret carefully and thoughtfully those fundamental laws, mechanical processes, generalizing procedures, and possibilities of practical applications which so definitely characterize mathematics as a field of study and endeavor. Thus, in addition to providing a general education for cultural background, the program for the professional preparation of teachers of mathematics should equip the teacher of mathematics with an integrated philosophy of education, a devotion to teaching as a profession, and a sense of responsibility for the contributions he will be expected to make in his chosen field of work. This body of professional knowledge should be provided through courses designed to acquaint the individual with the place and function of education in our social order, the interrelationships that exist between the various professions, and the manifold opportunities for service which present themselves to teachers; to build up "a sympathetic understanding of the mental, physical, and social characteristics of the children or adults to be taught";[2] and to provide "opportunities for acquiring a 'safety minimum of teaching skill' through observation, participation, and actual practice under supervision."[3]

Although the cultural background and the body of professional knowledge are essential elements of the program of professional preparation of teachers of mathematics, such a program must not overlook the fact that "sound scholarship is the fundamental qualification of the teacher." This scholarship, however, should be *relevant* to the problem of teaching. From the point of view of the teacher of mathematics, what materials provide opportunity for development of relevant scholarship? It is high time that we who are interested in mathematics should realize that the scholarship that is of service in the advancement of mathematical thought and research is not, in every case, the same as that which can be of service in the education of adolescents and immature thinkers. It is almost proverbial to state that "he who learns that he may know and he that learns that he may teach are standing in quite different mental attitudes."[4]

The specialist in mathematics has need of a synthetic type of scholarship in which he seeks mastery not only of the fundamentals of mathematical thought but also of a closely interwoven chain of logic and of methods of making deductions and implications. His only need for the analytic type of scholarship is as a tool to be used in the aid of mathematical research. He is not necessarily concerned with questions as to what is the place of mathematics in the educational program or as to what practical value is the study of mathematics. The principal question which concerns him is "What are the implications?" not "What are the applications?" He is the producer of mathematics.

On the other hand, the teacher of mathematics is the seller of mathematics. He is the one who must convince the consumer and, incidentally, through the medium of efficient service, secure consumers. He is constantly bombarded with questions as to the value of mathematics to the individual and as a significant element in the program of general education. He must be primarily interested in seeking the answer to the question "What are the applications?" and secondarily interested in the question "What are the implications?" Thus, in the training of the teacher of mathematics, the analytic type of scholarship must be emphasized. He

[2] E. S. Evenden, "Summary and Interpretation of the National Survey of the Education of Teachers," *Office of Education, Bulletin 1933, No. 10*, VI (Washington: Government Printing Office, 1935), p. 93.

[3] *Ibid.*, p. 94.

[4] H. S. Tarbell, "Report of the Sub-committee on the Training of Teachers," Proceedings of the National Educational Association, 1895, p 240.

must be taught to evaluate mathematics in the light of its orientation in the history of civilization, its contribution to the present social order, and its possible relation to future progress. Furthermore, since it is to be his responsibility to later impart mathematical information to the immature learner, the prospective teacher of mathematics should not only strive for proficient mastery of his subject, but also make every effort to be conscious of the processes by which he acquires that mastery. He should pause at significant points for moments of reflection in which he would strive to analyze the learning processes involved and evaluate the materials studied. Masterful scholarship must be constantly emphasized, and, for the teacher, this scholarship must be defined in terms of mastery of fundamentals. There is undoubtedly a desirable scale of such fundamentals to be mastered, which depends upon whether the prospective teacher is to teach in the elementary school, junior high school, senior high school, or junior college. Such a scale of fundamentals should constitute a body of minimum essentials, so to speak, satisfactory mastery of which would be required of every prospective teacher of mathematics. Furthermore, each such teacher should be encouraged to strive for a certain synthetic proficiency in some chosen line of mathematical endeavor to serve as a reserve of information which he might frequently use as an aid to individual exploration in the unknown realms of mathematical knowledge or in the expanding domain of significant applications of mathematical principles and techniques.

In an educational program designed to be of maximum benefit to the members of a democratic social order no body of knowledge is of value except in terms of its value to the individual and to the social order. Thus

> *. . . teacher education must be judged by its effects on teachers; teachers must be judged by their effects on pupils; and pupils must be judged by their competence as members of a democratic social order.*[5]

Masterful scholarship in a body of relevant knowledge is an absolute essential for effective teaching but it must be supplemented by a proficiency in the use of efficient techniques of instruction. Neither should be overemphasized but a proper balance should be maintained throughout the preparation program. We do not want teachers of mathematics to be "teachers who have nothing to teach," neither do we want them to be "mere purveyors of knowledge and promoters of skill."[6] In the light of the demands made on the teacher as he strives to function in our modern program of public education, it seems to be rather axiomatic to state that

> *. . . the teacher needs to have over and beyond the essential competency of one who would guide and direct children in a field of study, proficiency in:* (*a*) *knowledge of the place and function of his work in the entire program of the school;* (*b*) *specific objectives of the fields of instruction he is teaching;* (*c*) *selection of materials, textbooks, supplementary reading, equipment, etc.;* (*d*) *use and interpretation of various types of tests—standardized and non-standardized, essay and objective* [*factual and functional, furthermore, this must imply a familiarity with the fundamental problems and philosophy of significant evaluation of instruction*]; (*e*) *methods of adapting assignments to groups of children;* (*f*) *selecting varying methods of teaching;* (*g*) *guiding and directing activities of pupils both curricular and extracurricular;* (*h*) *knowledge of special problems of content and method in teaching his fields of specialization.*[7]

[5] K. W. Bigelow, "The American Council's Commission on Teacher Training," *North Central Association Quarterly*, XIII (October, 1938), 200.

[6] National Survey of the Education of Teachers, *Teacher Education Curricula*, Bulletin 1933, No. 10, III (Washington: Government Printing Office), p. 96.

[7] *Ibid.*, p. 97.

We might list the above as the necessary technical equipment of the efficient teacher. As a highly desirable, but not absolutely necessary, part of the program of professional education of prospective teachers we should list skill in the use and interpretation of the techniques of educational research and experimentation. The individual teacher should be equipped to read, interpret, and evaluate the published results of experimental investigation and make use of significant findings in the improvement of his own teaching procedures. It is also very desirable that he be equipped to pursue scientific investigations within his own program and to interpret intelligently his findings for the benefit of others.

I am announcing nothing new when I state that things are not as they should be in the teaching of mathematics in the United States. Such studies as the National Survey of the Education of Teachers[8] and the Report of the Committee of the North Central Association on "Subject Matter Preparation of Secondary School Teachers"[9] tell us that

> . . . changes in the nature of the high school student body, the expansion and diversification of the program of studies, the new responsibilities for guidance, the incorporation of student activities into the curriculum itself, the tendency toward curricular integration, the marked changes in educational objectives, all together indicate a revolution in secondary education to which the subject matter preparation of teachers has certainly not been adjusted with sufficient rapidity or appropriateness.[10]

The Mathematical Association of America and the National Council of Teachers of Mathematics have made notable efforts through their committees and publications to improve a deplorable situation.[11] The fact remains, however, that the teaching of mathematics is hardly an established profession. There are individuals teaching mathematics who have neither majored nor minored in the field, they have no interests there and can do no more than merely skim along on a surface of imperfect superficialities. They know nothing of the values of mathematics nor of its possibilities of integrated development. On the other hand, there are many teachers of mathematics whose program of preparation has been full as far as mathematics is concerned, but they are unhappy and inefficient in their work because they are having to teach one, two, maybe three or more other subjects. Their program of preparation should have been broad enough to have provided for preparation to teach in at least one additional field. There are other teachers who know their subject but are actually driving young students away from further study of mathematics by their inability, ofttimes unwillingness, to recognize student difficulties and weaknesses. They have no professional attitude that impels them to forget their "superior knowledge" and come down on the level of the immature student and patiently guide him through a labyrinth of pitfalls and encourage him to become interested in further exploration.

There is also the attitude of the decreasing value of mathematics that is prevalent in the minds of many administrators, educationists, and public laymen today. As long as this attitude prevails we can expect the employment of mathematics teachers, the assignment of mathematics classes, and even the inclusion of mathematics in the educational program

[8] *Ibid.*, Vol. I–VI.

[9] Final Report on the Committee on "Subject Matter Preparation of Secondary School Teachers," *North Central Association Quarterly*, XII (April, 1938), 439–539.

[10] *Ibid.*, p. 440.

[11] In addition to the publication of articles and discussions in *The American Mathematical Monthly* and *The Mathematics Teacher* there should be mentioned the recommendations by the Committee on Mathematical Requirements in their 1923 report on "The Reorganization of Mathematics in Secondary Education;" by the Commission on the Training and Utilization of Advanced Students of Mathematics in their 1935 report on "The Training of Teachers of Mathematics;" and the present work of the Joint Commission on "The Place of Mathematics in Secondary Education."

to be on a rather unstable professional basis.

The patterns for certification of teachers used by state departments of education and the various accrediting agencies contribute to the general chaotic condition that exists. No general agreement seems to be in evidence as to what constitutes adequate preparation for teaching any given subject, with possibly three exceptions.

What can be done to bring us out of this unfortunate situation? We can continue to sit at our desks or talk among ourselves and criticize the educationist and the citizen at large for their lack of understanding and appreciation of mathematics. We must remember, however, that while we are in our backyard slinging mud, so to speak, at the wall of misunderstanding that stands between us and the mathematical layman, particularly, the curriculum maker, the educational psychologist, the administrator, and their educationist cohorts, they are in their backyard doing the same thing. We shall never get anywhere that way. We can, each of us at our respective levels of mathematical instruction, continue to criticize the poor preparation on the level below us and wonder what is to happen to mathematics and mathematical instruction if such conditions continue to exist. Again we shall never get anywhere by that process. We need some concerted effort to attack the fundamentals of the entire problem and to attempt to construct and define a true profession of teaching of mathematics. To quote from Professor Kempner:

> ... tendencies of the last generation should have made it frightfully clear to all of us that the whole of mathematics, from the grades through high school, college, graduate school, and beyond, form an indivisible unit. Whatever harms mathematics at one level, harms it at all levels; whatever benefits it at one level benefits it at all levels.
>
> It is a good sign that we are beginning to realize two facts:
>
> First: We are dealing with a situation for which we mathematicians carry a greater share of blame than we have been ready to admit.
>
> Second: It can be remedied only by well-directed concerted action of the mathematics teachers of the country, acting as a whole, grade school and high school teachers, principals, college and university teachers, and graduate faculties.[12]

In conclusion I wish to present five proposals for positive concerted action designed to define the teaching of mathematics as a true profession and to outline a functional program for the professional preparation of teachers of mathematics.

Proposal 1. We should, with strong conviction among ourselves, recognize the teaching of mathematics as a significant profession. This implies that we accept the fact that there is a distinction to be drawn between the education of the specialist and that of the teacher; that this, however, is not a distinction in dignity or significance in the promotion of the program of mathematics, but a distinction in type of responsibility. All large departments of mathematics which have well-staffed divisions in analysis, algebra, geometry, etc., should also have a division devoted to the professional preparation of teachers. This division should be on professorial parity with all other divisions of the department. It should be the responsibility of this division to organize and promote a functional program for the preparation of teachers, to provide opportunity for apprentice teaching and its careful supervision, to encourage significant experimentation and investigation of problems related to the teaching of mathematics from the elementary school through the junior college, to serve as a sort of liaison division between the entire mathematics department and the school of education, or educational workers, for the promotion of better cooperation in the solution of important educational problems. The standards of research should be

[12] Aubrey Kempner, "The Mathematical Association and Mathematics in the Secondary School System," *The American Mathematical Monthly*, XLIV (December, 1937), 634, 635.

held just as high in this division as in any other division and graduation should be with a degree of value equal to that in any other division. From the B.A. to the Ph.D., inclusive, the degree should be one "in the Teaching of Mathematics," and this title should carry with it no stigma of inferior training. The department should recommend for teaching positions in the elementary school, high school, and junior college *only* those graduates from this division who had followed a program designed to prepare them for the specific position.

To give emphasis to this proposal may I quote from two recognized sources:

The problem of vested interests will loom large unless those with vested interests see clearly that their highest good is best realized by doing what will in the long run contribute the greatest values to the social process. The university teacher with his special field of knowledge must not fail to see that the general excellence of preparation of high school teachers will be important to him in the line of his speciality. He must be quite ready to consider different types of courses from those which he would plan and develop in the training of prospective specialists in his own field.

We cannot prepare prospective high school teachers properly if we give them simply the same intellectual food as we place before those who are becoming specialists in particular fields. In what way the several programs should differ one from another will have to be determined by careful analysis. But the specialist must be prepared, on his part, to modify his point of view in the light of the information which is or may become available.[13]

It would seem to me to be necessary to develop a new type of mathematical professorship, one definitely devoted to this problem [the supervision and training of teachers], to be filled only by a man recognized by educators as qualified in educational principles and also recognized by mathematicians as well trained in mathematics beyond the grade of the courses under his supervision.

....

The creation of such a position in every large graduate school and the enforcement of such requirements for practice teaching and for discussion of problems of teaching on all students who aspire to teach in lower divisions of colleges would seem to me to be a first step in the solution of the whole problem of mathematical instruction in this country. It would give us what we have not had, a body of men adequately trained in mathematics who are devoting their thoughts, at least in considerable part, to the principles of education. It would give us soon a body of young teachers in the lower divisions of universities who would have at least some ideas on why and how they are teaching their subjects. This in turn would affect the teaching in high schools, since the teaching processes of lower-division courses through which prospective high-school teachers pass would be done with more regard to educational ideas.[14]

All departments of mathematics should recognize and accept the responsibility of providing a significant program for the professional preparation of teachers of mathematics. As a corollary, no School of Education or Teachers College should pretend to train teachers of mathematics unless opportunity is provided for significant mastery of subject matter fundamentals.

Proposal 2. The National Council of Teachers of Mathematics should supplement the fine work it is already doing by a deliberate, well-planned, prolonged program characterized by continuity of effort and designed to build up a truly professional attitude in the minds of all teachers of mathematics in the elementary school, junior high school, and senior high school.

Proposal 3. The Mathematical Association of America should supplement its fine work with a similar program for the teachers of mathematics in the junior colleges and in the lower divisions of our senior colleges and large universities. Furthermore the Association should encourage experimentation and investigation on problems related to the teaching of mathematics on this level by offering opportunity for publication of significant results in the *Monthly*. It is to be hoped that

[13] R. D. Carmichael, "Implications for the Learned Societies," *North Central Association Quarterly*, XIII (October, 1938), 215.

[14] E. R. Hedrick, "Desirable Cooperation Between Educationists and Mathematicians," *School and Society*, XXXVI (December 17, 1932), 775–776.

these are two of the purposes of the newly established department on *Mathematical Education*.

Proposal 4. The Association and Council, with the sympathetic support and advisory council of the Society, should unite their efforts to plan a cooperative program of mathematical education designed to sell mathematics as a significant subject and the teaching of mathematics as a worthy profession to the public at large and the educationist in particular, especially the curriculum makers and the administrators in our elementary and high schools.

Proposal 5. A Cooperative Committee should be appointed to study the whole program of mathematical education and the preparation of teachers of mathematics. Mathematics, as a subject in the curriculum of our elementary and high schools, has probably suffered more severe criticism than any other one subject. Some of this criticism has been justified because we, who are workers in the field, have allowed ourselves to assume a somewhat traditional complacency concerning the value of mathematics as an instructional medium and its place in the program of secondary education. Some of the criticism is unjustified because a few educationists have become overenthusiastic in their interest in certain educational fads that are more temporal than lasting in their values. They fail to think in terms of ultimate values and are content to skim along on imperfect superficialities that seem to give immediate satisfaction.

This Cooperative Committee should have as its chairman one who is a member of the National Council of Teachers of Mathematics, the Mathematical Association of America, and the American Mathematical Society. He should be an individual whose training justifies and whose interests impel him to follow the activities of each group and to contribute to their publications. He should have given evidence of his intense interest in mathematics as a subject and in the professional preparation of teachers of mathematics. The membership of this committee should be made up of one member from each of the following groups: American Mathematical Society, Mathematical Association of America, National Council of Teachers of Mathematics, Society for Curriculum Study, Progressive Education Association, Association for Childhood Education. In addition there should be an Educational Psychologist, and one member to represent accrediting and certification agencies.

This committee should have the power and the facilities to define the program of mathematical education in its relation to the entire program of public education and also to *then* formulate a program for the better preparation of teachers.

These recommendations have emphasized three things: (1) conviction among ourselves of the dignity and value of the teaching of mathematics as a profession; (2) desire and willingness to cooperate in the furtherance of a real professional program; (3) continuity of effort in the promotion of this program. Conviction, cooperation, continuity, upon these three concepts hangs the future of mathematics as a subject in the educational program of the United States and the respect of the teaching of mathematics as a worthy profession in which to serve one's fellowmen.

Educational Purposes

THE events of recent years call for an exploration and restatement of educational purposes and obligations, and make it evident that adjustment to contemporary conditions and opportunities is imperative. If educators are to make wide and real the reach of their theory and practice in answering this summons, they must step over the boundaries drawn by their profession and consider the unity of things. Yet they are at the same time compelled, by the nature of their obligations, to hold fast to those values of education which endure amid the changes and exigencies of society.—Reported from the *Unique Function of Education in American Democracy.*

Mathematics in the Training of Arithmetic Teachers*

By R. L. Morton
Ohio University, Athens, Ohio

As a rather lengthy introduction, I should like to relate to you an account of a recent effort to solve a problem having to do with the interest rate which is equivalent to a given carrying charge in installment buying and to make the various solutions which were proposed intelligible to teachers of arithmetic in the upper grades. The problem is this:

On an order for $24.50, Sears Roebuck and Company require a down payment of $3.00 and $4.00 per month for 6 months, thus imposing a carrying charge of $2.50. From the customer's point of view, this carrying charge is equivalent to what interest rate?

First Solution. Since the customer defers the payment of $21.50, he has the use of this amount for one month. Also, he has the use of $17.50 for one month, $13.50 for one month, $9.50 for one month, $5.50 for one month, and $1.50 for one month. In all, he has the equivalent of the use of $69.00 for one month, or $5.75 for a year. Since the customer pays $2.50 for the use of $5.75 for a year, from his point of view the carrying charge is equivalent to an interest rate of 43%, for $2.50 ÷ $5.75 = .43.

Second Solution. Since the payment of $21.50 is spread over 6 months, the monthly payments on the order are one-sixth of $21.50, or $3.58⅓ each, the remainder of the payment of $4.00 being interest. The first of these payments is owed for one month, the second for two months, the third for three months, the fourth for four months, the fifth for five months, and the sixth for six months. This is equivalent to borrowing $3.58⅓ for 21 months, or for 1¾ years. One per cent of $3.58⅓ for 1¾ years is 1¾ × $.0358⅓, or $.0627⅓. But the actual charge is $2.50. Since $2.50 ÷ $.0627⅓ = 40, the carrying charge is equivalent to an interest rate of 40%. This solution is given in a well known series of arithmetic textbooks.

Third Solution. When the down payment has been made and the carrying charge has been added to the balance, the customer owes $24.00. Then, the customer owes $24.00 for one month, $20.00 for one month, $16.00 for one month, $12.00 for one month, $8.00 for one month, and $4.00 for one month. This is equivalent to $84.00 for one month, or $7.00 for a year. Since $2.50 is 36% of $7.00, this is equivalent to an interest rate of 36%. This solution is given in at least two other well known series of arithmetic textbooks.

An examination of the second and the third solution leads to the conclusion that the only important difference between them is the fact that the carrying charge is not included in finding the amount due month by month in the former but is in the latter. This gives larger amounts due each month in the second solution and, therefore, a lower interest rate. There is no important difference between saying that the customer owes $24.00 for one month, $20.00 for one month, etc., and saying that he owes $4.00 for one month, $4.00 for two months, etc. One gives $84.00 for one month, as we have seen, and the other gives $4.00 for 21 months. Of course, $84.00 for one month is equivalent to $4.00 for 21 months.

I am sure that you have already concluded that neither solution is a mathematically sound solution. But the problem for the teacher of arithmetic in the upper grades is not solved by providing a solution which is mathematically sound for such a solution is beyond the mathematical level of the course in these grades. The teacher must provide an *approximate* solution, rather than an exact one, and must use the method provided by the textbook in use by his class, or select another method which may be found in other well established texts. If he is to choose between the two solutions which have just been discussed, he must decide whether the carrying charge is properly added to the balance due before the amount owed month by month is determined. Let us consider a fourth solution.

* An address delivered at the joint meeting of the National Council of Teachers of Mathematics and the Mathematical Association of America, Williamsburg, Virginia, December 30, 1938.

Fourth Solution. The problem is one of finding the rate which makes \$4.00 monthly for 6 months have a present value of \$21.50, or \$1.00 monthly for 6 months have a present value of \$5.375. We must solve the equation, $\$5.375 = \frac{1-(1+i)^{-6}}{i}$, for i. To save time and labor, we may turn to present value tables. We find the present value of 1 per period for 6 periods to be 5.41719144 at 3% and 5.32855302 for $3\frac{1}{2}$%. Interpolating for 5.375, 3.24% is obtained. Multiplying 3.24% by 12 and rounding to the nearest whole per cent, we get 39%. This is seen to agree more closely with the rate obtained from the second solution than with the rate obtained from the third solution.

Fifth solution. The fourth solution, which involves finding the rate at which a series of periodic payments will have a given present value, certainly is sounder than either of the preceding solutions, but it has one possible fault. That fault lies in the fact that the formula or the tables which the solution uses are built upon an assumption of compound interest. But since installment buying usually involves only a short period of time, 6 months in our illustration, possibly simple interest should be assumed. Hence, a fifth solution. If the simple interest rate which makes \$4.00 monthly for 6 months have a present value of \$21.50 is calculated, it is found to be 36% to the nearest whole per cent. This is seen to be the same rate as that obtained by the use of the third of the preceding solutions. It is, in fact, essentially the same solution.

It is neither my purpose nor my responsibility at the present time to answer the question as to whether compound interest or simple interest should be assumed in setting up a solution to this problem. I am aware of the fact that the finance company which has a constant turnover of large sums may operate on an assumption of compound interest while the customer, whose investments are seldom compounded more often than every 3 or 6 months, may prefer to treat the problem as one in which simple interest would be assumed. But I have another concern and that concern, briefly, is this: It is impossible for the rank and file of arithmetic teachers to participate intelligently in a discussion of the various solutions which have been proposed simply because of their very limited experiences with mathematics beyond arithmetic and the elements of algebra.

I could continue at some length to recite to you the difficulties which teachers of arithmetic experience because they lack a subject matter background to reinforce them as they try to understand and then to teach certain topics in this subject. The need for additional experience in mathematics becomes so obvious to one who works with teachers and prospective teachers that it is very difficult to understand why the matter should be debated at all.

Those who are assembled here probably agree with what I am saying about the need for further experience with mathematics. However, when one undertakes to state what should be included in the teacher's courses in mathematics and how the content should be organized, he is sure to run into opposition. Nevertheless, I should like to make a few observations.

At the beginning, I should like to be able to assume that each of the young persons who present themselves to teacher-training institutions has had at least one year of algebra and one year of geometry in high school. However, this is not the case. I have analyzed the high school records of 2734 persons who received baccalaureate degrees from Ohio University in the six-year period beginning January 1, 1931, and ending December 31, 1936. This was a part of a larger study having to do with the place of mathematics in the college curricula of these Ohio University graduates. I found that 95 per cent of these 2734 graduates received credit for algebra studied in high school; that the number of units of credit earned in this subject ranged from one-half unit to three and one-third units; that the outstanding mode is one unit, earned by 45 per cent of the entire group; that one and one-half units were earned by nearly 38 per cent of the group; and that two and one-half units were earned by nearly 12 per cent of the group. In addition to these, there

were a few whose transcripts of secondary credits showed simply "mathematics," "general mathematics," and the like. Including these, and doing a little guessing, we arrive at a figure of 97 per cent who, apparently, have had one unit or more of high school algebra.

The geometry picture is not so bright. Only 86 per cent of these 2734 baccalaureate graduates had had geometry in high school. The typical individual had one year in this subject. My first proposal, then, is that

All students in teacher-training institutions who have not received credit in high school for a minimum of one year each in algebra and geometry should be required to take these subjects in college and should receive college credit for them.

What additional experience in mathematics should prospective teachers of arithmetic receive in college? Here, again, we must speak in terms of a minimum standard for, obviously, there is no maximum, although there may quite well be an optimum. The program of courses for teacher preparation as it is typically organized includes many courses from many and varied fields, so many indeed that it is futile to expect that room will be made for very many semester-hours of work in mathematics. I should like to see the minimum set at not fewer than 10 semester-hours for I find it difficult to see how a defensible minimum program can be organized so as to require less time. If the number of hours is very greatly reduced there is not only the probability that vitally important matters will have to be neglected but also the danger that under the pressure of time limitations the teaching will degenerate into mere drill and the learning into rote learning. Others believe that a defensible minimum program can be organized in 8 semester-hours; still others say that 6 will be sufficient. I know of at least one institution in which it was thought that very ample provision was being made for mathematics in the preparation of elementary teachers by setting up a single three-hour course. And, as you well know, there are many such programs in which no provision whatever is made for mathematics. This matter of semester-hours could be discussed at much greater length but I must hasten. I shall terminate the discussion by offering as a second proposal the following:

The minimum experience in mathematics of college grade for prospective teachers of arithmetic should be organized in a year's work of 6 to 10 semester-hours.

What should be the content of such a year's course? I think that you will agree with me that there are a number of items in the usual courses in Freshman mathematics, courses in algebra, trigonometry, and plane analytic geometry, which should be included but that in addition to these, room should be found for items from the mathematics of finance, items from both the differential and the integral calculus, and possibly items from astronomy. If you agree, you will conclude at once that the usual introductory college courses in mathematics as they are organized for students in the College of Liberal Arts or for students whose major interests lie in the fields of science or eingineering will not be satisfactory. The prospective elementary school teacher would have to choose between getting the requisite number of hours but failing to include all of the topics or getting all of topics but finding it necessary to take far more than 10 semester-hours. Hence my third proposal:

The college mathematics experience of prospective teachers of arithmetic should be secured through the medium of courses especially organized for this purpose.

I have no doubt that such a special course would be a very desirable course for others than prospective teachers to take. So far as I have suggested it, there is no reason to believe that it is better adapted to the needs of prospective elementary teachers than to the needs of other young persons who wish to devote only 6 or 8 or 10 semester-hours to the study of mathematics in college. In other

words, I am pleading for a special course for prospective elementary teachers not only because such a special course will be better adapted to their needs than will the more or less traditional college courses but also because I believe that in general a brief course in college mathematics should be a special course.

Unfortunately for this point of view, there are in this country many persons in positions of influence who are violently and unalterably opposed to all such special courses in mathematics. Such courses sound to them to much like "general mathematics," a term for which they seem to have conceived a violent dislike. Just recently, I learned of a department of mathematics in a good-sized university which is presided over by a chairman who will have no general mathematics or other special courses and who batters down all opposition with very loud and very fluent, although not particularly eloquent or very logical denunciation. His department offers a 10-hour course in Freshman mathematics, of which 3 hours are devoted to algebra, 2 hours to plane trigonometry, and 5 hours to plane analytic geometry. He asserts, again violently, that regardless of whether the student is to work out a major in mathematics for the B.A. degree, or is to use mathematics as a part of his program in engineering, or is to take a little mathematics as part of a liberal education, or is to study mathematics as a part of his background for elementary teaching, he should pursue these particular Freshman courses. Members of his own staff disagree with him but quite without avail.

It would seem that in this particular university (and I refer to it because it is more or less typical of a group) a more valuable and more interesting course of 10 semester-hours could by provided for prospective elementary teachers by reducing markedly the amount of time devoted to analytic geometry and introducing material from the mathematics of finance and elementary calculus. Furthermore, such changes in the course should give the student a more comprehensive idea of the varied nature of mathematics and of its wide usefulness in solving problems which are incident to our complex civilization.

The success of such a course in mathematics will depend to no small extent upon the experience and native equipment of the teacher. The mere fact that the teacher has been a competent student of mathematics and has a Ph.D. in the subject from a recognized institution is not enough to qualify him to give this course for prospective elementary teachers, whatever we may say about his qualifications for teaching other groups. In addition to possessing a knowledge of mathematics, he must be willing to break away from a traditional organization and work out an organization designed for a new purpose, he must be interested in collecting a rich assortment of applications in order that the topics of the course may be interesting and meaningful, and he must be cognizant of and sympathetic with the life ambitions of his students. In short, this is my fourth proposal:

The teacher of the college mathematics course which is designed for prospective elementary teachers should be a special selection for this purpose; there are many teachers of college mathematics who will not qualify.

Those who teach mathematics have, for many years, been deploring the apparent decline in interest in this subject. They say that those in administrative positions in the public schools supported and sometimes stimulated by professors of education, have failed to recognize the worth of this discipline and have gradually permitted it to be supplanted by other subjects, notably those in the field of the social studies and those involving manual activity. The school administrators and the professors of education, on the other hand, claim to have just reason for criticizing the offerings and the methods of the teachers of mathematics. It seems to me that there are many problems dealing

with the place of mathematics in the curriculum, with the organization of courses, and with the teaching of this subject which will not be solved until they have been attacked cooperatively by both mathematicians and educators. The specific issue with which we are here concerned —mathematics in the training of arithmetic teachers—will not be disposed of satisfactorily except through the medium of a friendly council in which members of both of these groups participate. Can the mathematician eat the salt of the educator? Can the educator break bread with the mathematician? And so, finally, my fifth proposal:

A college course in mathematics designed for the professional preparation of teachers of arithmetic should be organized cooperatively by a group composed of educators and mathematicians. Such a course should be planned as a six-semester-hour course with prescribed additions for making it an eight-hour course or a ten-hour course.

Mathematics in Industrial Chemical Research*

By W. W. Heckert
Wilmington, Delaware

As soon as chemists began to make quantitative studies and to look for mathematical relationships among their data they began to make progress at a greatly accelerated rate. This trend toward quantitative treatment has continued. Some of the most fundamental problems of the science, such as those of chemical affinity and reaction velocity, have now begun to respond to mathematical treatment. This trend has made itself felt in the industrial field. It is expected to continue.

Mathematics is used in industrial chemical laboratories (1) in the planning of research, (2) in the formulation of quantitative relationships which permit the testing of *ideas,* (3) in the development of testing methods for the evaluation of new *chemical products,* (4) in the interpretation and presentation of experimental data, (5) in engineering phases relating to the design and construction of plant equipment, and (6) in routine chemical calculations and the preparation of cost estimates. The speaker will confine his attention to the first four of these uses.

The degree to which chemical problems now respond to mathematical treatment is illustrated by slides showing how the equilibrium condition between ethylene and water vapor can be calculated from specific heat and other thermodynamic data. Examples drawn from the field of rayon research are presented to illustrate other uses for mathematics in industrial chemical research. The importance of rapid, approximate methods for calculation is stressed, since the mathematical approach must always compete on a dollars-and-cents basis with the empirical method.

* An abstract of a paper read before The National Council of Teachers of Mathematics at their summer meeting in New York City on June 27, 1938.

Some Preliminary Considerations Relating to Arithmetic in the Senior High School*

By H. E. Benz
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This paper is concerned with arithmetic as a high school subject. Before proceeding with the argument, it might be well to agree on definitions of terms. For purposes of this discussion, by arithmetic we mean that branch of mathematics which is concerned with numbers, and their application to the quantitative situations of everyday life. Perhaps we will obtain a better impression of what is meant, if we define arithmetic as that branch of mathematics which is generally taught in our elementary schools. However, the term should be delimited further by confining it to those aspects of the subject which are relatively free from algebraic notation, algebraic concepts, and algebraic procedures. At once we are compelled to recognize that under close scrutiny the boundary between arithmetic and algebra disappears. One definition of algebra is "generalized arithmetic." Perhaps the distinction which is intended here is best brought out by reference to the fact that most persons who are now over 30 years of age studied something which they called arithmetic when they were in the elementary school. It was quite free from such things as x's, equations, negative numbers, coefficients, binomials, etc. Then in the ninth grade such persons made the acquaintance of a new subject, which bore the name of algebra. The definition of arithmetic implied in this paper is that which prevailed in the elementary school of 25 years ago, so far as the scope of the subject-matter is concerned.

One further item of terminology requires clarification. The term "high school" as used here refers to grades 9, 10, 11, and 12. This is not to ignore the movement for the reorganization of the school system, nor to question the importance of the junior high school movement. Parenthetically, it may be noted that more than half of the high schools in the United States are of the 4-year-senior-high school type. It may further be noted that in the case of only one series of arithmetic texts have the author and publishers seen fit to wind up the series with grade 6. However, for purposes of this discussion, the precise point at which the elementary school is separated from the high school is of no particular consequence. But the term "high school" will be used to refer to those grades above the eighth.

The purpose of this paper is to examine certain considerations which may have a bearing on the question of teaching a course in arithmetic in the high school. It is presumed that such a course is intended for the general curriculum. Courses in arithmetic are now offered in various vocational curriculums, but with these we are not concerned at the moment.

Clearly, no discussion of the place of arithmetic in the high school should take place without reference to the objectives of secondary education. Lists of such objectives exist in bewildering confusion. We shall attempt to simplify the matter by referring to only one, the famous seven objectives listed by the Committee on the Reorganization of Secondary Education, and sometimes incorrectly referred to as the "seven cardinal principles." Before making specific applications, perhaps we can agree that these are objectives of *secondary education*. Some of them are no doubt important as objectives of elementary education also, but it must be assumed that the secondary school is to attempt to realize these objectives, whether or not they have been worked on in the elemen-

* Read before the Christmas meeting of the National Council of Teachers of Mathematics at Williamsburg, Va. on December 30, 1938.

tary school, with due regard for overlapping and repetition.

We shall refer to each objective briefly, in some cases quoting from the report, and attempt in the space of a few lines to indicate the relationship between the acceptance of the objective and the thesis being presented in this paper. The first one is "*Health.*" Certain proponents of high school mathematics teaching have attempted to set forth in detail certain items in mathematics which might have implications from the standpoint of the attainment of the health objective. Curiously, although the implication is that these offer a reason for pursuing the usual high school mathematics program, the specific items mentioned usually fall in the field of arithmetic. The second objective is "*Command of Fundamental Processes.*" The committee says, "the facility that a child of twelve or fourteen may acquire in the use of these tools is not sufficient for the needs of modern life." The third objective is "*Worthy Home Membership.*" Perhaps a better understanding of the considerations incident to the intelligent management of the domestic budget would contribute to the attainment of this objective. This topic certainly is primarily arithmetical. The fourth objective is "*Vocation.*" Since the importance of arithmetic in the training for certain vocations is accepted, and since this paper is concerned with non-vocational education primarily, we shall pass this by. The fifth objective is "*Civic Education.*" Whole areas of civic education which involve arithmetical computations and arithmetical insight have been left relatively untouched. One need only think of the large place that taxes play in our governmental relationships. The sixth objective is "*Worthy Use of Leisure.*" The failure of many newspapers to use decimal points when listing batting averages of baseball players seems to indicate the need for a better use of arithmetic in connection with leisure time activities. The last objective is "*Ethical Character.*" The relationships between this objective and certain applications of arithmetic are too obvious to require mention, but one illustration may not be out of order. How about the efforts of many persons to remedy the deficiencies of a meager income by playing slot machines?

However, the purpose of this brief reference to one of the standard lists of objectives, together with the illustrations of certain applications, is not so much to establish our case as to set up a framework of reference, within which the question of the importance of any proposed material for the high school curriculum must be discussed.

Before leaving this question of the objectives of secondary education certain viewpoints relative to the high school which are now quite generally accepted, and which are assumed in this discussion should be mentioned. In the first place, the social utility theory of education is postulated. This of course need not necessarily mean a "bread and butter" utilitarianism. The term must be used in a broad sense. For clarity, it may be set off against a point of view which relies on formal discipline, or on some vague mystical thing called "culture," usually not defined, or on that other remnant of the prescientific age of curriculum building, "learning for its own sake." The assumption here is that every single item in the secondary school curriculum must "pay its way" in terms of concrete usefulness in the life of the learner.

With this background, we may turn to an important question. Who and what brought up this question of arithmetic in the high school? Certainly, we may take the point of view that the advocate of any subject or section of a subject for inclusion into the high school program must present evidence to indicate a deficiency in the present program, in terms of the life needs of pupils, or adults. Perhaps 99% of the adults whom we meet in our every day contacts have taken the work in arithmetic which is now universally re-

quired in our elementary schools. Is the arithmetical equipment of these persons adequate? Are they able to make the maximally effective adjustment to their environment, in terms of any conceivable definition of the good life? Pending the scientific research necessary to a defensible answer to this question, we may rest our case on the observations of everyday life. There seems ample reason for making the generalization that most adults today are woefully deficient in arithmetic.

We detect several different types of deficiency. In the first place, they are deficient in the fundamentals of computation. Who has not seen retail clerks become confused over a column of figures to be added? Many sales slips contain errors, most of which go uncorrected because housewives do not feel certain enough of their own accuracy to demand corrections, or because they find the experience of addition so painful even in the privacy of their own kitchens, that they seldom add them. Every bridge foursome contains at least one person who dislikes to keep the score because of his ineptitude, and at least one more who will gladly keep it but who can not be trusted to keep it correctly. Gasoline stations post quantity prices, and now have installed computing pumps because attendants make mistakes and customers would rather trust such figures than those they arrive at by their own efforts. Any number of illustrations of what we may call the computational illiteracy of the average adult could be mentioned. Perhaps introspection and recall will furnish others.

In addition to a lack of ability to use arithmetic effectively in the sense of mastery of the fundamental computations, most persons are not too familiar with the *applications* of arithmetic to the affairs of everyday life. The inability of elementary school pupils to solve verbal problems is well known to teachers of the subject. The inability of adults to apply arithmetic to concrete situations is comparable. Most owners of automobiles simply don't know how to go about the matter of determining the cost per mile of operating a machine. The question of the relative cost of owning or renting a home is often answered in terms of other considerations because most persons have not learned the techniques of arithmetic which are involved. Many other illustrations of the same inability of the average individual to use his arithmetic to help him solve the bewildering and perplexing problems of life, could be suggested. Any normal person's own experience will offer additional evidence of a need for attention to this phase of education.

The third phase of this problem of the inadequacy of the equipment of normal adults has to do with those applications which have not been treated at all in the past. The arithmetic of installment purchasing offers a case in point. This is a relatively new phenomenon in American life. While most modern arithmetic texts make a gesture in the direction of a recognition of its presence, a thorough study of the topic is not possible under present circumstances. The amazing growth of the small-loan business indicates another weakness in our arithmetic instruction in the past. Thousands of persons are building homes with the aid of loans under the F. H. A. plans, and most of them have had inadequate instruction in those phases of arithmetic which would enable them to compute their payments, or their interest rates. Understanding and appreciation of the effect of various proposed schemes for taxation involve a type of arithmetical approach which is not possible to those whose study of arithmetic has been limited to the conventional elementary school program. Other illustrations could be suggested, but time does not permit. But let it be remembered again, that the mathematics involved in the applications here suggested is strictly arithmetical in nature. No doubt many of these topics are illuminated by a study of algebra, but the material is primarily arithmetic.

Much has been written recently about

informational arithmetic. Not all who discuss the term use it in exactly the same sense. It usually refers to those concepts and ideas which are associated with the study of arithmetic, but the study of which does not involve computation or the exercise of the manipulative skills of arithmetic. It is well-known among psychologists that one of the most common impediments to the mastery of concepts and ideas is a lack of mental maturity. The difficulty of a concept increases as it becomes more abstract. The more difficult a concept is the longer its presentation should be postponed. Thus we are led to raise the question whether whatever we are expected to try to accomplish under this general heading of "informational arithmetic" might not be accomplished more economically and more effectively if we postponed it until the high school. Accurate information based on actual experimentation is lacking, but casual observation leads us to believe that postponement would be desirable.

The idea of teaching arithmetic as a high school subject is not entirely new. In fact, at the present time thousands of high schools are doing it. A comprehensive and thorough survey of the situation relative to arithmetic courses in the high schools should be made in the near future, and perhaps such a survey will soon materialize. A fragmentary survey indicates that out of a sampling of 50 representative high schools, somewhat more than half offered some course called arithmetic. By far the most of these courses are known as business arithmetic or commercial arithmetic. However, many of them originally intended to contribute toward a vocational objective, are open as electives to students in the general curriculum. This leads us into an examination of the various possibilities. Some courses in high school arithmetic are best characterized by the phrase, "a rehash of the elementary program." They cover substantially the same ground that is covered in the elementary school. The teacher gives tests at the beginning of the semester, finds that the pupils are woefully deficient, and begins by staging a review of the elementary field. If the teacher is conscientious and persistent, and believes that this relearning must precede additional work, the semester is usually over by the time the task has been completed. Some courses attempt to extend the elementary school work in terms of speed, accuracy, and difficulty. These courses again cover about the same ground that the elementary school does, so far as the nature of the material is concerned. However, they attempt to give the pupil a better mastery of arithmetic than the objective which the elementary school sets up. Columns are longer, division examples have longer divisors, fractions are more complicated. Verbal problems have more steps. A minimum amount of attention is paid to social utility. Much of this point of view prevailed in early courses in so-called "rapid calculation." The teacher attempted to achieve a higher level of accuracy than the elementary school ever did, even at times pursuing that intriguing will-of-the-wisp, "100 percent accuracy." Then there have been the special courses, above referred to, which were intended for certain vocational curriculums.

It may not be amiss at this time to present, somewhat cautiously and with many reservations, a program for the course here under consideration. It would seem that the first step would be to define and particularize the aims of secondary education. Perhaps the list referred to above would be a good one to use. However, each of these seven objectives would need to be made much more specific. Under each one would be placed a list of more specific objectives, somewhat after the manner of Bobbitt's formulation.

The next step might be the sorting out of those specific objectives which might be realized by a course in arithmetic. These could be classified and arranged according to categories that might have some significance in terms of the subject.

The final step would be the organization of the course, based on the attainment of the objectives desired. It should be noted that the direction of procedure is from the objectives to the desired course. A more common way in the past has been to set up the course, and then attempt to find as many objectives for it as possible. This method has led to a vast amount of fruitless rationalization and to the defense of much worthless subject matter.

However, a more specific program is involved. Certain other steps would need to be taken. One of the first difficulties would be the determination of the speed and accuracy in the fundamental operations which is necessary. An amazing amount of research has been done on the elementary school arithmetic curriculum, but no good answer has ever been found to the question of just what speed and accuracy the world demands for efficient living. Inasmuch as the proposed high school course will have to deal with this problem, we should make every effort to determine just how much time should be given to the improvement of these aspects. However, in addition to studying the arithmetical needs of persons as they are found in the world now, an effort should be made to make some judgments relative to the amount of arithmetic which people would use if they knew enough of it. This point has often been emphasized by curriculum builders, and need not detain us longer.

We should also need to know more accurately than we now know, the degree of speed and accuracy possessed at present by children when they finish the elementary school, when they finish the high school, and that possessed by adults of varying levels of competence in life. Finally, we shall need more information on what degree of speed and accuracy can be expected as the elementary school refines and improves its objectives and its procedures. Evidence at present available indicates a loss of efficiency in grades 7 and 8. Maintenance programs will do much to reduce that loss, once these grades can be brought to accept responsibility for maintaining skills which are taught in lower grades. Furthermore, there are reasons for thinking that the new emphasis on meaning in arithmetic, with the more intelligent rationalization of the computational processes and the substitution of insight for at least some of the drill which has characterized instruction in our subject in the past, will result in a better mastery of mechanics.

In addition, we shall need much information about the every day activities of people, especially as those activities involve situations the manipulation or comprehension of which involves those concepts which are primarily arithmetical in nature.

At this point a word of explanation should be made relative to the point of view implied in this paper. Much reference has been made to the activities of adults. It should not be understood that there is a lack of appreciation of the immediate, day-to-day needs of children for arithmetic, or that those needs are to be ignored. The fact that children every day have use for arithmetical procedures and concepts is well-known to observers of child life. The point of view that children should be bottled up in school, taught the activities of adults, and prepared for adult life, with complete disregard for their immediate interests and needs, is fortunately a thing of the past in modern schools. At the same time we can not ignore the importance of the fact that the child will some day be an adult. Sometimes we have been so preoccupied with child development that we have forgotten the end product of that development, namely, adulthood. And we dare not ask the public to support an elaborate system of education which ignores preparation for adult life. Our schools must teach the child to use arithmetic in the situations of life, as well as the situations of childhood.

At this point it may be well to set down, more or less systematically, the

informational arithmetic. Not all who discuss the term use it in exactly the same sense. It usually refers to those concepts and ideas which are associated with the study of arithmetic, but the study of which does not involve computation or the exercise of the manipulative skills of arithmetic. It is well-known among psychologists that one of the most common impediments to the mastery of concepts and ideas is a lack of mental maturity. The difficulty of a concept increases as it becomes more abstract. The more difficult a concept is the longer its presentation should be postponed. Thus we are led to raise the question whether whatever we are expected to try to accomplish under this general heading of "informational arithmetic" might not be accomplished more economically and more effectively if we postponed it until the high school. Accurate information based on actual experimentation is lacking, but casual observation leads us to believe that postponement would be desirable.

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At this point it may be well to set down, more or less systematically, the

reasons for the proposed course in high school arithmetic. First, the elementary school cannot complete the job which it has laid down for itself. That is not a reflection on its competence. It is based on examination of the facts. In the second place, approved statements of the objectives of secondary education include specific objectives which can be met only by a course in arithmetic. Anyone interested in this may examine the Bobbitt list, above referred to, and select those objectives which involve arithmetic. These can be met by a course in arithmetic in the high school. They are not being met, and probably can not be met by the usual elementary school course. Third, the present day courses in mathematics in the high school do not and can not meet these objectives. They may meet other equally important objectives very well, but they do not devote enough attention to the subject matter of arithmetic to realize the values indicated in this discussion. Fourth, observation and the results of tests indicate that many high school children do not have an adequate mastery of the arithmetic which is taught in the elementary school. And this can be accepted as true whether the definition is in terms of life needs or in terms of the accepted objectives of the elementary school. These may, of course, coincide, but perhaps they do not always do so. Finally, the proposed course in arithmetic in the high school is necessary because there are many uses of arithmetic which cannot be taught at all in the elementary school. These uses are quite as important as any of the other material which is taught in the secondary school in connection with other subjects. The important limiting factor which prevents teaching this material in the elementary school, assuming that the additional time necessary could be made available, is the lack of a mental and social maturity on the part of the pupils sufficient to guarantee understanding. The important limiting factor which prevents teaching this material in the present high school mathematics courses is the fact that they at present have other objectives which necessitate the use of other subject matter.

This matter of the possibility of teaching the material here under consideration in other courses in high school mathematics deserves further attention. Superficially, there is no reason why this could not be done. But it is not being done and it probably will not be done. There are several reasons for this. It seems out of order to expect teachers of courses which are called algebra or geometry to teach these topics. It should be remembered that we are referring to material which is not algebraic in its nature. Clearly the teacher who introduced any considerable amount of subject matter into a course, which did not logically belong there in terms of the content as implied in the title, could be accused of a mild form of educational fraud. This is not to argue that we should maintain rigid subject matter divisions but rather that we should permit the title of a course to describe the content as accurately as is convenient. There remains the possibility of teaching this material in courses listed under the title of *general mathematics*. And there certainly are many such courses in high schools today. However, for the most part, those sections of the courses designed for grade nine or grade ten are singularly free from arithmetical topics. They consist for the most part of correlations of algebra, geometry and trigonometry. The material represented in this discussion simply is not to be found. There are several possible explanations for this state of affairs. One is to be found in our traditional college entrance requirements. Another may be found in the fact that these courses are worked out, and textbooks for them are written by persons who think in terms of the division between elementary school arithmetic and high school mathematics which was referred to early in this paper. These persons simply do not believe in teaching arithmetic in the high school. In

this connection it is to be noted that the course in business arithmetic is more often taught by someone in the commerce department than someone in the mathematics department.

Certain objections to the proposed course present themselves. Each of these objections will be the subject for comment as it is suggested. *First,* such a course would simply be a re-hash of the elementary school course. There is danger of this. However, it need not be. The type of material contemplated is for the most part new, not now taught in the elementary school. Some of the elementary school material would be moved up into the high school course. *Second,* it is not properly secondary school material; arithmetic belongs in the elementary school. This is the voice of tradition, and has nothing to do with the scientific study of grade placement. Many people think elementary algebra and plane geometry are not college material, but at least one university is teaching them. *Third,* children would not be interested in such material. About this we know very little. True, it is proposed to move into the high school certain topics in which children manifest very little interest at the present time. It is believed that that lack of interest is in part a reflection of social immaturity. Many of the proposed new topics deal with material which is a part of the environment of every normal person. There seems to be no good reason for pressing this objection in advance of trial of the material. *Fourth,* such a course would overlap the present algebra and geometry courses. It is presumed that competent curriculum specialists can prevent this from happening. *Fifth,* the necessary instructional instruments are not available. This is for the most part true. However, several textbook intended for use in such a course have appeared recently, and some of them contain important fractions of the material here contemplated. When the principle is once accepted, and teachers begin to demand instructional materials, they will be made available. Furthermore, competent teachers with a real sense of the importance of their material will not let the lack of textbooks block their efforts to provide worthwhile educational experiences for pupils. *Finally,* it is agreed that those teachers most likely to be called on to offer such a course, will, because of their training and prejudices, not be sympathetic to the point of view, hence will not do a good job. Unfortunately, that may be true. There is a real danger that brief trial may result in the development of a negative attitude toward the whole problem. If first experiences are bad, the course may fall into disrepute, its objectives fail to be realized, and the experiment abandoned. To the extent that this is due to incompetence, stubborness, or narrowness of educational vision, it need not concern us further.

No doubt at this point there is some demand for a more accurate description of the proposed course, or perhaps for a brief syllabus of it. None will be presented. It is hoped that this discussion has presented the general point of view, and has indicated, although in very general terms, the ground to be covered and the objectives involved. More precise delineation and accurate description of the course can not be undertaken at this time. However, if we accept the point of view, and appreciate the importance of the objectives, the description of the subject matter to be covered will not be difficult and in time progress can be made.

To CAPTURE the citadel of a child's mind through love and sympathy; to lead pupils toward higher ideals of life and duty; to establish closer relations between home and school, and state; to exalt purity of life and conduct; to strengthen the moral tone of the community; to make good men and women; to establish and dignify the profession of teaching; to make education attractive; to magnify the state; to meet the need for educated citizenship; such is the exalted mission of the teacher. —HON. CHARLES W. SKINNER.

Modern Curriculum Problems in the Teaching of Mathematics in Secondary Schools

By W. D. Reeve
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I. Problems Relating to the Pupil

Nature of the High School Population

The United States has more children above fourteen years of age in school than all the other countries of the world. In many communities, we have sixty per cent and in a few cases as high as ninety per cent of the ten million pupils of eligible age in school. High school enrollment has grown five times as fast as the population in general. According to Douglass,

The estimated enrollments in public secondary schools have increased as follows:

1900	519,000
1910	915,000
1920	2,200,000
1930	4,400,000
1935	6,100,000

In 1900, 56.3% of the 519,000 pupils in public high schools were studying algebra. This means that practically all freshmen were studying algebra and that there was perhaps one in four or one in three of the other classes also studying second year algebra. In 1935 instead of 56.3%, there was slightly more than 25%, or one in four.

In 1900, 27.4% of all high school pupils were studying geometry. This means that practically all students studied geometry in either the junior or senior year. In 1935, less than 15% of the pupils in high school were studying either plane or solid geometry.

While the *percentage* of high school pupils studying algebra and geometry has decreased, the *number* has increased very greatly. In 1900, only 291,000 pupils were enrolled in algebra, but in 1935 more than a million and a half pupils were so enrolled. Comparable figures for geometry reveal a like growth. This is attributable to the tendency to make secondary education universal. It is of great significance to those interested in the teaching of mathematics that high school enrollments have just about reached the saturation point.[1]

The American public school system is unique in world history. Its pupils are certainly the most schooled of any group in the world, but are they as well educated? We no longer have so select a group of pupils in the secondary schools as formerly because, as can be seen from the table above, the high school population has increased more than 1200% in the past thirty-nine years. However, the high school population still approximates the best fifty per cent of American intelligence. Just imagine then how some of our teaching problems would be complicated if everybody were in school. As it is, the teachers insist that the problem of the slow-moving pupil is almost impossible to solve.

The Problem of Individual Differences

In spite of what was said above, the differences among high school pupils not only in native ability but in experiences and in interest are still so great that they challenge the best thinking we can do if the greatest good is to be accomplished with everybody concerned. How can we take care of these great differences as we find them in the high school today? The whole question of training for leadership and intelligent followership is involved as it should be in a democracy. At the present time, we are not giving the best training for leadership. The most retarded pupil in our secondary schools is the gifted pupil or the pupil of scholarly mind. He is the one who should be a leader in his generation.

According to the *New York Sun* of March 4th last year, in a recent experiment carried on jointly by Dr. Leta S. Hollingworth of Teachers College with the Board of Education in New York City, it was disclosed that "In a little more than

[1] Douglass, Harl R., "Let's Face the Facts," The Mathematics Teacher, Vol. 30, p. 56, February, 1937.

one term of instruction, the fifty bright pupils enrolled in the experimental (Terman) classes of the Speyer School gained from three to four semesters of knowledge in geography and science." There is an enormous range even among gifted pupils.

In discussing pupils' potentialities, Dr. Hollingworth points to the wide range of I.Q. (intelligence quotient) found among the fifty Terman children. These statistics were announced shortly after the school was organized in 1936, but in the present report, Dr. Hollingworth gives them in greater detail.

While an I.Q. of 130, or 30 per cent above average was the standard minimum for admittance to the bright classes of the Speyer School, the range among the fifty pupils is "enormous"—as much, Dr. Hollingworth points out, as between a child of average ability and an imbecile. Thus one boy had an I.Q. of 194 plus, or 64 points above the standard minimum of 130. This is the more remarkable, she points out, since persons with 130 or higher I.Q. represent one in a hundred of the community.

The youngster with the I.Q. of 194 is 9 years and 6 months old. Intellectually he is capable of good work in the top grade of a senior high school. "But it would be absurd to place him there" because of his physical and social immaturity, Dr. Hollingworth contends.

Not only are these bright children capable of doing school work far above the grade in which they were classified in their regular schools, but many of them actually have absorbed knowledge far above that required. As a group the Terman pupils—as the experimental bright ones are officially designated—took tests in the ordinary school subjects a year ago. It was found that, as a group, they had been graded one grade too low for their achievements. In potential ability they had been graded two grades too low.

"They actually already know (not merely had the capacity of knowing) what they were being taught and also what was being taught in a whole grade ahead of their status," says Dr. Hollingworth in summarizing her findings.

Of the fifty children enrolled in the Speyer School only two had been accurately graded in their home schools, even though almost all of them had been skipped at least once and were already young for their grade.

"To skip them to the point where they would have had appropriate mental work would be dislocate them very badly as regards other phases of their development," Dr. Hollingworth adds. "The principals of the schools of origin, are, therefore, not to be reproached for malgrading within the existing organization. Indeed, there are on file many letters from principals nominating pupils from their schools for whom at present they cannot provide. It is rather a modification of organization that is to be sought for these pupils."

Professor Douglass maintains that

> The central idea of re-vamping the curriculum to meet individual differences is not to be attained by petty tinkering with traditional courses nor by introducing a variety of new subjects. Surely by now we have recognized the fallacy of attempting to solve the problem by the impractical short-cut method of diverting a considerable proportion into vocational courses.
>
> The solution lies, instead, in developing more than one version of the chief fields of subject matter. We need, for the less verbal minded and those who will never go to college, a completely different set of courses in English, in science, in the social studies, and in mathematics, and perhaps also in music and art. Few teachers possess the time, background, and imagination necessary to produce these courses. Textbooks are not yet available. Yet we can expect in the near future that progress in certain centers such as Oakland, Baltimore, and Denver will be made in that direction. What is really needed is a grant of a large sum of money by some foundation to subsidize a project requiring the full time for several years of a few outstanding teachers in each field who would be charged with the task of developing such courses and suitable textbooks.[2]

[2] Douglass, Harl R., "Can we Re-vamp the High School Curriculum to Fit the Needs of Today," Baltimore Bulletin of Education, Vol. XIV, No. 3, September–October 1936, p. 56.

Professor Douglass also says:

> The necessity of adapting instruction and instructional materials to the needs, interests and capacities of the less able youth now coming in constantly increasing numbers into the secondary school, should not be permitted to divert attention from the important responsibility of making the most of genius and unusual abilities of the ablest minority. Efforts should not only be made to identify those with whom nature has been most generous, but especial provision should be made for them. Especially adapted courses of study should be provided in the larger schools for them. In all schools especial efforts to accelerate their growth up to capacity should be made by the classroom teacher.
>
> In the interest of the nation and in protection of society's stake in the educational enterprise, at least a goodly share of the more brilliant youth should be directed into government service and into business leadership, and appropriate training provided. As rapidly as possible, the organization of public service should be so modified as to reserve for properly selected and trained men and women the very large majority of permanent public positions and the tenure and salaries adjusted so as to be attractive to the best of American youth.[3]

Traditional Plans for Solving Our Difficulties

Many plans for helping us to solve some of our pressing curriculum problems have been suggested by educational leaders. Among them are

1. The Dalton Plan with its contract idea. It has really worked better in England and is better known there than it is here in the United States where it originated.
2. The Winnetka Plan. This scheme placed a high value on "individualized instruction."
3. Homogeneous classification of pupils according to ability. This is the first step toward individualized instruction where the pupils are grouped into gifted, normal, and slow-moving groups. Through tests of one kind or another, we are now able to classify children into ability groups and yet in many schools where this is done, teachers proceed as though no differences in ability, experience, or interest exist.
4. Large Size Classes. This movement never made much headway and is now passing out.
5. Supervised Study. This plan had its good features, but in many places supervisors have been very unpopular lately. The point is that supervised study is *study* or it is not. If the teacher does it, it is not study in so far as the pupil is concerned.
6. The Project Method. Supplementary projects should always have a place, but in so many places the project method has resulted in "all project and no arithmetic" to such a degree that it has lost much of its value for mathematics. In some of the recent studies conducted in progressive schools of the effect of the project on learning content material, we find that there is a substantial gain in most subjects, but a clear loss in arithmetic.[4]
7. One radical experiment[5] proposes to show that pupils who have not studied arithmetic succeed better than those who have had a regular course in it. While it is just possible that such a condition might exist, I do not believe that the facts will substantiate the claims in the experiment referred to above.
8. Child Centered School. The extreme to which this scheme may go in spite

[3] Douglass, Harl R., Monograph on Secondary Education, Preliminary Draft, pp. 2–21, American Youth Commission of the American Council on Education, 744 Jackson Place, Washington, D. C., 1936.

[4] Breed, F. S., "The Liberal Group in Education," *Educational Administration and Supervision*, 22: 322, May, 1936. A paper read in part at the convention of the Department of Superintendence of the NEA, St. Louis, Feb. 24, 1936, in a debate on the topic "The Activity Program Is Inadequate in Arithmetic."

[5] Benezet, L. P., "The Story of an Experiment," *The Journal of the National Education Association*, November, 1935, pp. 241–244, December, 1935, pp. 301–303, and January, 1936, pp. 7–8.

of the fine idea back of it is illustrated by a cartoon recently appearing in the *Saturday Evening Post* where a small boy quailing before his teacher makes the appeal, "Teacher, do we have to do what we want to today?"

President Hutchins of the University of Chicago said recently

> The child-centered school may be attractive to the child, and no doubt is useful as a place in which the little ones may release their inhibitions and hence behave better at home. But educators cannot permit the students to dictate the course of study unless they are prepared to confess that they are nothing but chaperones, supervising an aimless trial and error process which is chiefly valuable because it keeps young people from doing something worse.[6]

9. The Integrated Program. I have written more fully about this plan elsewhere[7] but it ought to be said here that the idea is sound. The word "integrated" is unfortunate because it is a psychological term and really refers to what goes on in the pupil. We ought to use the word "correlated" but of course that is out of date. One thing is certain, we cannot integrate what we do not know. And before be try to correlate mathematics with art, music or even science, to say nothing of all the other great fields of knowledge, we had better learn first how to correlate informal geometry with arithmetic and algebra and to show the general interrelations in our own field first. So much depends upon who does the correlating that we had better be cautious about any promiscuous mixing of subject matter by people who are not properly trained to do it. To err here would be as bad as entrusting some important psychological considerations to an amateur psychologist.

[6] Hutchins, R. M., "The Higher Learning in America," Yale University Press, 1936, p. 70.

[7] Reeve, W. D., "Mathematics and the Integrated Program," THE MATHEMATICS TEACHER Vol. 30, p. 155, April, 1937.

All of the above plans at their best are no doubt sound. At least they have sound ideas back of them and I do not wish to disparage them unduly, but I do wish to sound a note of warning to those who regard them as panaceas. They are not. What we should do as educational leaders is to conserve the best in each.

If one not only had control over the fourth dimension but could also travel fast enough so that he could peer into ten thousand classrooms tomorrow, I dare say he would find in the vast majority of such classes no one set plan of teaching in use. Rather, I think he would find each teacher doing the thing most evidently suitable to the situation as it occurs at the time—sometimes almost lecturing, other times board work, again seat work or perhaps class discussion with an occasional period given over to diagnostic testing and the like. Often the pupils themselves select the procedure by which they will learn most easily and most economically. The best teachers do not confine themselves to one method at a time. If they did, the monotony would be unendurable. They try all of these schemes sooner or later and in the end retain what is good in each.

The development of psychology from a speculative philosophy to an empirical science has affected both the content material and methods of instructions in our schools. The pupil, his capacities, and his needs have come in for an amount of attention never before accorded him. In other words, we are attempting to get the pupil's point of view.

It is our duty, therefore, to study our own situation and to see to what extent we are making progress. We should see to it in the future that the child knows not only what he does, but why he does it. There is no reason, as far as I can see, why the processes of mathematics should not be rationalized just as far as time permits. There is far more danger that the subject will become tiresome and cease to function in the lives of the pupils because it is

taught too mechanically than that the various processes will ever be over-rationalized.

The Evils of Mass Education

The trouble with us in this country is that we have adapted mathematics to mass education with the natural result that we have produced a lowering of scholarship—"a leveling down instead of a leveling up" as Professor Bagley has often pointed out. Most, if not all, of our examinations in recent years have reduced standards until what we used to expect of pupils in the ninth grade we are glad to have them attain in the eleventh. To be sure, examinations should be made simpler and easier for certain types of pupils, but not for all!

The Awful Effect of Emotionalized Attitudes

Not much real thinking is done in this country by large numbers of our people in politics, religion, or even in mathematics. Much that is being done is accomplished by developing certain emotionalized attitudes. If this is true, how do they reinforce or interfere with us as teachers? Why should so many people hate mathematics, particularly algebra? The answers might be obtained by asking such people why they hate it, but I doubt it. Is the hatred the result of an emotional complex? If so, why?

A woman who sat near me at a dinner recently said, "How in the world can you ever teach mathematics so as to make it interesting to children?" I told her that it was very easy to do, but I do not think she believed me. Many of our students in Teachers College who feel that they must take the course in Educational Statistics do so with fear and trembling because of the mathematical requirements which, after all, are not severe. Little more than a working knowledge of arithmetic is required.[8]

The first opportunity to get a favorable attitude toward mathematics comes to the teachers of arithmetic. Such teachers should not drive their pupils to hate mathematics by stupid teaching.

It is fairly obvious that where we are killing interest in algebra today we are doing it by stupid teaching and over-emphasizing obsolete and formal processes like the factoring of such expressions as x^2+5x+6. Even some of our newest textbooks contain page after page of seventeenth and eighteenth century algebra. The solution to such a problem is not to eliminate algebra as some educators would have us believe—that would be stupid. The answer is to be found in reorganizing mathematics all along the line so as to include fundamental material that has a high practical as well as cultural value, and in teaching it as though salvation depended upon it.

Important Questions

There are certain important questions that need to be answered:

1. What pupils should be encouraged to study mathematics beyond the ninth year?
2. What part of the secondary course in mathematics shall be required and what part elective?
3. How important is the I.Q. in the pursuit of mathematics? Is it true that a pupil with an I.Q. of 100 cannot learn algebra? Of course, the answer depends upon what kind of algebra is meant and to some extent, upon the teacher.
4. What should be the nature and extent of the elective part of mathematics?
5. Why do pupils fail? Is it due to sheer inability to teach, faulty teaching, or lack of will to learn? Is it true that certain pupils "just can't learn mathematics"? If it is true, what can they learn?

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[8] Brown, Ralph. *Mathematical Difficulties of Students of Educational Statistics.* Bureau of Publications, Teachers College, Columbia University, 1933. See also Rogers, Charles F. "Arithmetic and Emotional Difficulties in Some University Students," *The Mathematics Teacher,* 30: 3–9, January, 1937.

6. What is the effect of our antiquated marking system on success in mathematics?

7. How do pupils learn most easily and most economically? Assuming, for example, that we should teach a pupil to factor "the difference of two squares," how long will it take a reasonably good teacher to teach it to a certain specified degree of mastery?

8. What has age to do with the problem of learning mathematics?

9. What has interest or lack of it to do with the learning of mathematics?

10. Shall we try to find out what slow pupils can learn regardless of its value or shall we try to ascertain whether they can learn some important mathematics concept previously agreed upon as worth teaching?

11. Is the present content of mathematics too hard or too easy?

12. What has speed or accuracy to do with the problem of learning in mathematics?

13. What mastery should we expect in mathematics? If we cannot expect complete mastery, what is a desirable objective?

14. What should we finally expect of a pupil well trained in high school mathematics?

15. How can we best evaluate our teaching?

These and many more questions need to be answered at once.

II. Problems Relating to the Teacher

Necessary Qualifications for Mathematics Teachers

The trend at present is toward higher qualifications for all teachers both in subject matter and along professional lines. In mathematics at the present time, the greatest need for some is a better knowledge of subject matter; for others it is more work in the foundations of education. One cannot teach what he does not know. However, mere knowing will not suffice. Educators should cooperate in this matter of "Teacher Training." Many teachers of mathematics do not even know the subjects they are expected to teach. Too many of our teachers have a weak academic background. We have an oversupply of teachers, but not of good ones. In this respect, our standards are far behind those of European countries. Almost anyone can teach mathematics in the United States. We now have teachers in some schools trying to teach trigonometry in the ninth year who have never studied the subject previously. The result is obvious. In some schools, a teacher may be given an algebra class merely because he happens to have a vacant period at the time the class is scheduled. It is an old story that the athletic coach is often given a class in mathematics to justify his employment in the school. We ought to be able presently to require a working knowledge of calculus of all prospective high school teachers. This subject is already required for the bachelor's degree of student teachers of mathematics in some of our American institutions, notably, at the University of Minnesota. At Teachers College, Columbia University, no one is given a diploma as Teacher of Mathematics or Supervisor of Mathematics who has not had a course in this subject.

The two articles in this issue by Professors Wren and Morton will be of interest to all those of us who are interested in seeing some new plans for the better training of mathematics teachers all along the line.

Why do teachers fail? Mainly because of a lack of personality, but there are many other reasons. Cannot these reasons be catalogued and cannot we, by studying them, improve the present situation? Can we, by some scheme or other, decrease considerably the number who seem unable to succeed? Doubtless fewer teachers would fail if they know more subject matter, but knowledge of subject matter alone will not save a teacher who is short on personality and the ability to under-

stand pupils sympathetically. However, it is not always safe to conclude that a teacher is an artist merely because he obtains good results on achievement tests. Some of the best drill masters in the world have been anything but inspiring in the classroom. Teachers may be born and not made, but surely something may be achieved by adequate training.

We have an insularity of mind among our teachers of mathematics and perhaps among other teachers as well. We are too nationalistic in our educational outlook. In mathematics education, for example, we are one year behind the European schools, and at the end of our secondary schools we are two years behind. By travel, study, and reading, our teachers should become broader in their outlook, but what is the best way to proceed? The reading habits of teachers need to be improved. They are not able to improve themselves enough in this respect, partly because they have too many classes to teach, and partly because they have not learned how to conserve their time for reading and study.

Mathematics teachers in particular are too often slaves of tradition. This can be remedied only by reading and study at home or at summer school, if the year-round school is not available.

Teachers, particularly those in the larger towns, do not have enough meetings of their own department and clubs are almost unknown in most towns. There is great need among teachers for group enthusiasm, group consciousness, group loyalty and group courage, not necessarily for political reasons, but merely so that they may know what is going on and how they can solve some of their common problems.

Another real problem is to secure better cooperation between the elementary teachers, the junior high school teachers, and those of the senior high school. At the present time, each group knows too little about the work of the others. There is no doubt that the attitude of senior high school teachers toward their colleagues in the junior high school has been a serious handicap to the mathematics program of the latter. This attitude may be explained, but the explanation does not justify the failure to cooperate.

The united effort of a homogeneous group of any sort has always proved to be one of the best methods of accomplishing great tasks. Many organizations of teachers since 1910 have been formed not only to advance their own personal interests, but for the purpose of improving the general educational situations. Witness the report of the National Committee on *The Reorganization of Mathematics in Secondary Education* and the recent Classical Investigation and now the new Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education."[9]

Teachers need a philosophy of education of their own—not a super-imposed one. It may be affected by the impacts of other minds but in the last analysis it must be their own. This they will not get as long as they are forced to follow a course of study which they have had no hand in making or with which they are not in sympathy. No matter how desirable a curriculum may be, we cannot expect it to succeed if the classroom teachers are not given a chance to help in its construction.

Many secondary mathematics teachers

[9] Report of the National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education*, The Mathematical Association of America, Inc., 1923. See also Parts I and II of A Preliminary Report by The Joint Commission of the Mathematical Association of America and The National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education." It is intended to publish the final report of this Commission as the Fourteenth Yearbook of The National Council of Teachers of Mathematics.

in this country do not even belong to the National Council of Teachers of Mathematics, the only organization in the country devoted *entirely* to their interests. Some do not even know that such an organization exists. The yearbooks of the National Council, a great source of inspiration and help to many teachers of mathematics, are not known to most teachers of mathematics and are not read by many who know that they exist. How can such apathy among teachers be removed? Every teacher of mathematics in the United States should be a member of the National Council. This membership costs only two dollars per year and entitles one to receive *The Mathematics Teacher*, the official journal, eight times per year.[10]

A new day in mathematics ought to bring forth a generation of teachers who can set up their own objectives, select the content material best fitted to their realization and, with proper methods of teaching, organize an evaluation program that will enable teachers to find out whether their aims can be realized.

This problem of securing better trained teachers for the schools is the hardest of all to solve. If we had a more cultured and highly capable group of teachers we should not be hearing so much about the problems relating to the pupil and to content. Obviously, professionally-minded teachers of ability would know very well what to do. Their standards would come from within, whence, after all, come the best standards. If we could only get educators generally to concentrate on this problem and cooperate in its solution, we should have less need for committees, commissions, pressure groups, and the like, trying to decide on what the curriculum ought to be expected to assimilate from it with profit.

[10] Teachers desiring to join the National Council should send two dollars to *The Mathematics Teacher*, 525 West 120th Street, New York City.

III. Problems Relating to Content

General Mathematics

It now appears that the future course in mathematics in many of the secondary schools of this country is to be *general mathematics* as well as the traditional courses in algebra, geometry, trigonometry, and the like which will be found in other schools. This is as it should be, but there are still many schools which do not appreciate the importance of this way of organizing and teaching mathematics. Moreover, there is good reason for believing that the high school course in mathematics should culminate in the calculus (not the old, but a new type) in the twelfth year. This is due to increasing pressure on the colleges to do work of a higher type demanded by the complexity of our present civilization. However, there are other types involving numerical calculation now being taught in the schools which should not be too much neglected by teachers who overemphasize the calculus. For example, approximate computation, statistics, and business arithmetic are relatively new topics coming into the senior high school that need further elaboration and emphasis. The final report of the Joint Commission referred to above will doubtless be helpful here.

The junior high school movement began to be an important factor in American education about 1915. This created a situation in the seventh, eighth, and ninth grades which was largely independent of the school situation as these grades were commonly organized. This new development afforded a chance for progress and we find considerable change made in the curriculum in some schools.

The change, however, has not been uniformly satisfactory even in the schools which go by the name of junior high schools. In mathematics, for example, we still find arithmetic occupying all of the time of the pupils in the seventh and eighth grades in many schools with a

sharp line of demarcation there followed by a year of traditional algebra. We are the only nation in the world of any consequence still giving eight years to arithmetic. A corresponding situation exists in other subjects as Professor Koos has pointed out.[11]

The senior high school course is now the least satisfactory part of the entire six years of the secondary school. We are generally agreed on the work of the junior high school in so far as content is concerned except that in some schools and in some textbooks we still retain too much obsolete and difficult material. It is fairly well agreed among the leaders both in mathematics and in secondary education that the course of the junior high school should be of a general nature[12] and that it should be required of everybody. In any case, then, whatever mathematics is taught in the senior high school can be made elective. However, it seems clear now that before we can decide what the mathematics course of the senior high school should be, we must first determine the content of the course in general education and then decide what the place of mathematics and the other great fields of knowledge should be. One difficulty confronting classroom teachers in secondary education is that educational leaders fail to agree. On the one hand, a man of the standing of Chancellor Chase of New York University says

> Economics, modern history, sociology, philosophy, basic ideas about the natural sciences, some knowledge of the fine arts, are today a far more necessary part of the equipment of the cultured layman than is the merely formal study of languages and mathematics. . . . The idea that liberal eduation should be a unity, based on some coherent philosophy of life has become obscured. Once it was a unity, centered about the classics. It gave to a few a type of culture that marked them off as a class apart. Now it must unify itself again, not about the classical cultures, but about the problems of contemporary life.[13]

On the other hand, Dr. Hutchins, the President of one of our greatest universities, says

> The tradition in which we live and which we must strive to help our students understand and clarify is hidden from out sight because of our own defective education. We are all the products of a system which knows not the classics and the liberal arts. There is every indication that the system is growing worse instead of better.
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> Every day brings us news of some educational invention designed to deprive the student of the last vestiges of his tools and to send him for his education helpless against the environment itself. The worst aspects of vocational education, progressive education, informational education, and character education arise from the abandonment of our tradition and the books and disciplines through which we know it.
>
> The arts central in education are grammar, rhetoric, logic and mathematics. The liberal arts are understood through books and books are understood through the liberal arts. The tradition is incorporated in great books. The teachers of English are the last defenders and exponents of these books and of the arts of language.[14]

The work of philosophers like Dewey, Kilpatrick and Bode has great influence on the classroom practice in America. They have glorified the American child, emphasized his possibilities and his rights and have demanded that the course of study be built up with these things in mind. Other leaders like Bagley, for example, without any intention of neglecting the child, remind us that the possibility of a more careful consideration of content material in curriculum construction must not be overlooked. It is this possible content material we need to examine, rather than to spend so much time on general methods of teaching.

Again, there are the sociologists who forcing the social utility issue in the wake of the greatly modified theory of mental discipline insist not only that the pupil

[11] Koos, L. V., *The Junior High School*, Ginn and Company, pp. 244–245.

[12] See Smith, David Eugene, and Reeve, W. D., *The Teaching of Junior High School Mathematics*, Ginn and Company, 1926.

[13] Chase, Harry Woodburn, *American Mercury*, November, 1934.

[14] N. Y. Times, January 31, 1937.

and his development be placed at the center of educational interest but also that all subject matter be modified and reorganized on the basis of its learning difficulty. As Professor Nutt has put it, "To inquire how the subject is getting on in the pupil, rather than how the pupil is getting on in the subject" indicates a very significant trend in modern teaching.

Most of the changes that have taken place in the curriculum are due to "external social forces" which have exerted an influence on the schools. Professor Judd says

> If one were disposed to be pessimistic, one would be tempted to use the terms that have frequently been used by critics and would say that fad after fad has been injected into the school program without adequate reason or justification.[15]

Little change has been affected through research. The three or four outstanding examples of recent reforms accomplished as a result of research are given by Professor Judd. He says

> Ayres changed at a single stroke the contents of the course in spelling. Laboratory investigations are directly responsible for the present emphasis on silent reading. Certain studies of the social demands for mathematics have been influential in modifying the amount and kind of mathematics taught in the schools.

Granted that we are able to set up a list of desirable objectives, that we choose the subject matter which seems best fitted to help us realize those objectives, that we have mastered the best known methods of teaching, and that we are well informed in the psychology of learning, how can we expect our work to be successful without an elaborate testing or evaluation program? Some of the objectives which we set up may be too difficult for a child at a given age to attain; others may be too easy. We know that the mastery of many of our pupils in some of the fundamentals is pitiful. We should have known that a long time ago if we had tested more. It is enough to say that the testing movement is in need of serious study. The emphasis just now needs to be put upon the use of tests for the improvement of instruction. The present abuses connected with the use of norms need to be corrected and it is our duty to make the testing program a vital and integral part of the entire problem of curriculum construction.

So much has been said and written concerning the examination evil that what I say may be nothing more than an unconscious repetition of what has already been said many times before, but it does seem to be to the point to say that whenever examinations, or rather the bare results of examinations, become the guiding principles for administrative action, we are getting dangerously close to doubtful practices and we do injustice to the pupils and teachers alike. There is little question that our system of examinations has led to a maximum amount of rote memory to the extent that many of our pupils have never discovered that any other method of acquiring knowledge or developing power was possible. A boy in a certain solid geometry class made what would ordinarily be called a perfect recitation on a solid geometry theorem and the teacher had assigned a mark of "excellent" on that recitation when some one raised a question concerning the figure on the board which, when answered by the boy, showed conclusively that the figure existed for him only in the plane of the blackboard. Knowing the history of that particular boy, I am sure that memory work was his only tool for passing the courses in mathematics, and he developed the habit largely through the influence of a consistent but unwise system of tutoring outside of class. The only remedy for such a practice in any system is to have the work of children supervised. They should be taught when it is best to memorize, and when it is best to study according to some judicious and well-thought-out plan.

The movement to reorganize the sub-

[15] Judd, C. H., "The Place of Research in a Program of Curriculum Development," *Journal of Educational Research*, 17: 313–325.

ject matter of mathematics and to improve its teaching is due to a number of influences. We may be sure, however, that the increased interest of the teachers of mathematics themselves has been due largely to a general dissatisfaction with the results shown by the testing movement. Not all of these results indicate the real value of the teaching that preceded the tests because a great deal depends upon whether the tests really serve as a check upon what had actually been taught. Moreover, some of the results of the tests seem to be very satisfactory, but we know that they are not always permanent. Much of the testing has been on mechanical processes, pupils fail to catch the spirit of the subject, and often are unable to apply what they have learned in advanced courses or in work that has a practical flavor. In other words, we often find that the pupils functional mathematical equipment is meager.

The growing conviction that mathematics should be reorganized, or let us say organized in better fashion, and that the teaching of the subject should be improved seems now to be quite general among the teachers themselves. For a long time, every thoughtful teacher has been doing his best to improve the unsatisfactory conditions in organization and methods. It is unfair to the teachers of mathematics in this country and aside from the facts to say that the improvements that have been made are due to the few laymen or "hobby-riding" enthusiasts who have been crying for reform no matter how important their contribution may be. It is only when the teachers themselves begin to discuss the cause of poor results and thus set up machinery for ameliorating the situation that we begin to make genuine progress.

As I have already implied, changes in the content or in its arrangement alone cannot produce any great improvement. The difficulty is deeper and includes the teaching side of the problem as well. No plan of curriculum construction or revision can succeed that does not at the same time take into account the best methods of improving teachers in service. The teachers always have the last say as to what shall be taught and how it shall be presented. We should have them represented at the conference table when curriculum problems are being discussed. This does not mean that the entire responsibility for such work should be shifted to the teachers. It does mean that teachers when they are competent should be a part of the entire curriculum building program.

Winter Geometry

The careless right geometry
Of trees against a hill of snow
Disarms the little math I know,
For the angle of branches against the sky
Informs the accomplished country eye
Of the kind of tree, the kind of fruit,
And the length, proportionate, of root.

This black and white geometry
Is something for the schools to teach—
As where the shadow falls of each
Branch and trunk, as the sun decrees,
On the hill that is tilted at x degrees;
Or the height of a pine whose radii
At its conic base will measure y.

This unperturbed geometry
Metes the arc that the Summer green
Will burden with solidity; and here is seen
What the rain and the snow and the restless weight
Of leaves will do to what grows straight
Up to the light—and how much sky
Will be lost to the view when a year is by.

—Ellen Acton in the *New York Times*, December 16, 1938.

The Efficiency of Certain Shapes in Nature and Technology*

A Suggested Unit of Instruction on Intuitive Solid Geometry

By May Hickey
Mississippi Delta State Teachers College, Cleveland, Mississippi

Introduction

Existing curricula in the junior high schools show a weakness in failing to provide pupils with adequate experiences in intuitive solid geometry. Those space problems that appear are usually of a static nature where a formula is used to obtain some geometrical property of the figure. The visual and tactual experience with solids and their properties, the manual construction of various figures, and the dynamic purpose of certain shapes are missing. The inclusion of these would infuse vitality and purpose into the curriculum and allow the subject of solid geometry to become effective and functional in the lives of pupils.

With the failure of the junior high school to provide this training—where properly it seems to belong—the senior high school could be expected to make partial amends to those pupils who continue their education. But the senior high school curriculum shows exactly the same weaknesses. Even the one subject, solid geometry, which it offers is made ineffective because of the meager and inadequate experiences which pupils have had with solids and spatial thinking. Whatever the benefits are that a pupil derives from the study of solid geometry as it is usually taught, the fact is that very few pupils elect to study the subject. The result is that secondary school education is at present contributing practically nothing toward developing in pupils adequate space notions and an intelligent appreciation of spatial relationships.

This state of affairs is not due to a lack of interest of pupils in solids and their properties and shapes, nor does it result from their lack of intelligence. Non-academic and average persons of a community recognize the important part that solids, and especially their shapes, play in biology as well as in modern designing of machinery and architecture and of almost every form of vehicles for transportation. In this instance the schools are their own impediment to educational progress: they harbor the classical notion that the application of geometry to practical ends is degrading. This is at variance with the observations that children enjoy labor where they have a purpose in their efforts and they stand unashamed before practicality.

From educational considerations the material in intuitive solid geometry seems most fitting to the eighth grade. The pupils of this grade are interested in experimental and manual methods which to them are essential to adequate proof. They want to make the solids and touch and look at them. They are not especially lazy but will do mathematics with much enthusiasm if they want to answer a question important to them. They only insist constantly on having a purpose in their labor. Whatever they study in intuitive solid geometry must provide them with such a purpose.

The following material, sketched in brief, is given as being of the nature described above, complementing partially the curriculum in intuitive geometry. It is recommended for instruction in the eighth grade.

In particular this material is of a nature that suggests vital questions whose answers the children are desirous and eager to find. It lends itself to various methods of attack by elementary mathematics in which the individual pupil's outlook plays

* An address given before the Mathematics Section, Mississippi Education Association, Jackson, Miss., April, 1937.

a part. It uses the scientific approach through the general setting of the problem to isolate special viewpoints, and emphasizes the approximate nature of mathematics. It allows of manual construction of solids and shapes and actual tactual experience with them. It is of much interest to the average person of the community and has an important bearing on present trends in machine designs and architecture. Finally it relates two important fields of human interest, biology and technology, and lends itself admirably to the unit method of instruction in socialized mathematics.

The form of the material is that of general questions or problems followed by one or more activities of children to answer certain aspects of them. Some suggestions are given for procedures in instruction and a brief bibliography for the teacher as references in the subject matter.

A Study of Shapes as Related to Economy

(1) If one has a certain length of fencing and wants to enclose a rectangular plot, how should he shape it in order to enclose the greatest possible area? How if the field were isosceles triangular in shape? How if it might be in any shape whatever? If a rectangular field is to have a given area, what shape gives the least perimeter?

Activity. By trying various shapes of rectangles of the same perimeter, computing their area, and arranging the results in tabular form, the pupils may conclude that the best shaped rectangle is a square. The class may divide into groups, each using a different perimeter to determine whether a square is the best rectangle for any perimeter. The same method applies to show that the isosceles triangle should be equilateral. That the shape for a given perimeter which gives the best area is a circle can be concluded by considering successively an equilateral triangle, a square, a regular hexagon, octagon, a 16-sided polygon, and a circle. The inverse problem may be solved in the same way.

Neat figures might be made to scale, cut from colored paper and pasted on a cardboard to form an attractive summary to the conclusions. The color scheme and the relative size of the figures, made to the same scale, should be chosen by the class, the best figures selected by the class, and a pupil who can print well chosen to make the necessary explanations on the poster.

The class may decide they would like to give an exhibit of their work in mechanical drawing as a possible conclusion to the unit.

A variety of other applications of the same principles can be studied in varying the conditions, as fencing only three sides of a field, or finding the least cost of fencing when that along one side is more expensive than along the others.

Activity. A purpose of windows is to admit light. The class may study the general shapes of windows in residences, office buildings, and churches and obtain by measurement the relative dimensions. Conclusions may be drawn with respect to the major function of windows and the adaptations made in designing them to meet the demands of utility and art in architecture. A poster showing the adaptations may be made.

(2) A boy is to race from a point on land to the edge of a rectangular swimming pool and thence to swim to a corner at the opposite end. For what point should he head at the edge of the swimming pool in order to complete the race in the shortest possible time?

Activity. One notes first that the shortest distance will not always give the shortest time to travel. Assuming reasonable rates of running and swimming, as well as certain distances, pupils can find by computations the time required for various landing places and conclude from a table of values the best point at which the boy should jump into the pool.

The problem may be varied to the following: If cows coming into an enclosure by one gate must walk to the straight edge of a pool for drinking and thence out

of the field by another gate on the same side of the pool, at what point at the edge of the pool should they tend to go for a drink in order to go out the second gate in the least possible time? Do cows seem to know the answer to this geometry problem?

Application. A light ray traveling from a point in air to one in glass always goes by the path to give the shortest time for travel. This is the only way light can travel. Earthquake and radio waves behave in the same way. If the media through which they travel vary continuously, these waves travel in curves. They always know exactly how to go in order to require the least time.

(3) According to Post Office regulations the combined length and girth of a parcel sent by parcel post must not exceed a certain length. What is this regulation? In what shape should a parcel be sent in the form of a rectangular solid with square ends to contain the greatest volume? What if the parcel is a right circular cylinder? What if it is a right circular cone?

Activity. Inquiry at the Post Office will give the postal regulation. By making various rectangular solids satisfying the given regulation the class will conclude that the greatest volume is obtained when the length is twice the width of a square end. The other shapes are determined in the same way. For the cylinder and cone the best volume is obtained when the height is π times the radius of the base. Pupils should construct to scale various solids of each type, comparing their shapes with that having the greatest volume. Such shapes can be easily made from thin cardboard and glue paper.

A selection of the shapes made by the class may be added to the exhibit of plane figures. Neatly lettered explanations on the surfaces as to dimensions and volume summarize the activity. Some of these may be given by the class as a contribution to a permanent class room collection of surface models.

(4) What shape of cylindrical tin can of one quart capacity will require the least amount of tin?

Activity. By experimentation and computation it may be shown that the relative dimensions for least surface of the can are height and diameter equal. Models of various cylinders of one quart capacity should be made and their shapes compared. These could be added to the collections of the other activities.

Activity. Groups from the class may make visits to a grocery store for measurements of height and diameter of cans of standard tinned foods, listing contents, capacity, weight and dimensions. In the class room ratios of the height to diameter may be computed to find how near the constructions are to the most economical. The adaptations of shapes of cans for other purposes than economy of material should be noted, such as the shape for convenience in shipping, the kind and shape of food canned, the quantity an average householder might want to buy in one can, and the price of this quantity. The class may write to one or more of the canning companies to ask why they selected cans of a particular shape. Investigation may be made of the best proportions for an ordinary gallon can without a top. Models of the well known standard cans might be made with explanations of the possible reasons for such a shape.

(5) What shape does a soap bubble or a balloon take if it floats in the air? Why does it take this shape?

Activity. The bubble contains a given amount of air. The investigation may be made to compare the surfaces of various rectangular solids with that of a sphere of the same volume. The conclusion is that the sphere has less surface than the rectangular solids that contain the same volume. The reason that a soap bubble or a balloon takes the spherical shape is that all *elastic* surfaces tend to stretch as little as possible; that is, their surfaces are as small as possible to enclose the given volume.

(6) What shape does a soap film take

when it is made as follows? A large circular wire ring is dipped into a soap solution and removed with a soap film across it. A smaller wire ring is brought up to touch the film across the large ring and is slowly pulled away at right angles. The soap film is stretched.

Activity. The experiment may be performed at home and in the school room. The surface of the stretched soap film at any instant has a definite shape (a catenoid of revolution) depending on the relative sizes of the rings and the distance between them. It will be found that the surface becomes longer and thinner as the distance between the rings increases until the elastic film can do less stretching by taking another shape. The film then slowly separates, part retreats to the larger ring, the other to the smaller ring. The surface is now the sum of the two areas of the rings.

Models of soap or wax can be made showing the shape of the surface at various instants. Properly labeled with an explanatory card they may be added to the collection of surfaces of previous activities.

The experiment may be varied by choosing different sizes of rings or pulling the circles apart in a direction not at right angles to each other.

(7) What adaptation in shape do seeds or insects make for floating in the air for a considerable time without internal effort? Why is a parachute shaped as it is?

Activity. Observation of such forms as milkweed seeds or spider webs that float in the air or the wings of butterflies or dragonflies reveals that large surface for the weight is the adaptation.

(8) Study the shape of a water bug that skims over the surface of a calm pool. What peculiarity of shape seems to fit it for running over the surface?

Activity. An experiment with a dry sewing needle on the surface of a pan of water shows that if laid carefully on the surface it will float. A body of the same weight spherical in shape will sink. The tendency of the liquid molecules is to hold together. This resistance to separation on the surface is called surface tension. The weight of the needle distributed along so great a distance is not enough to break the tension of the water. Surface tension depends on length. The water bug's characteristic shape is to gain length. Small animals which depend on travel over the surface of water are adapted in structure so that as much of the body as is possible is distributed in length and often the body is covered with very light hairs.

(9) What is the characteristic shape of plants and animals which must stay afloat to live?

Activity. Observation will reveal that they have some buoying adaptation that keeps them afloat. Large air-filled bulbs or bladders or hollow stems buoy up some plants. The small animals must have a form that gives them the greatest volume for their weight. As a small animal grows, its volume must increase for its weight or it will sink. Some fish are buoyed up so that they remain within a certain range of depth in the ocean. Their air-filled bladders may adjust themselves to depths within that range but above it, the fish will rise to the surface and below it, will sink to the bottom.

(10) What is meant by *streamlining*? What adaptive purpose does it serve?

Activity. Study the shapes of swift birds to see what adaptations their bodies have made to gain speed in flight. What is the characteristic form of whales, sharks, trout and other swift fish? The shape in general is a long somewhat narrow body, bluntly rounded at the head and tapering gradually to the tail. This shape is best for the animal to glide through the fluid with the least amount of disturbance to the fluid. Study the shape of the hull of an ocean liner to see how it is adapted to speed. Collect some bullets to compare their shape with that of a shark. Does the shape of a bullet seem to be best for speed? Obtain pictures or models of a submarine torpedo. Does it seem well shaped for its purpose?

Obtain pictures and toy models of streamlined trains, automobiles, and airplanes and study their shapes. What is the purpose of a speed boat? Obtain models of some. Why is their hull not shaped like that of an ocean liner? Study the shape of the Transpacific Air Clippers.

Summary. The portion of the material collected as pictures, drawings, or sketches of solids may be arranged in a booklet where explanations and conclusions of investigations may be written. The title of the booklet and the design for its cover may be chosen by the class. The booklet may serve as a place for recording measured and computed data. In it should be written the story of the development of designs of cars, planes and ships and the historical accounts of certain famous scientists and engineers who have helped improve the designs of industry.

A class contest may be held at the close of the unit in which the members of the class bring their best illustration of adaptation in shape among forms of nature or designs in industry and explain how its shape is well fitted to the purpose of the body. Various examples may be found in the community such as leaves, winged insects, sponges, toy models, and honeycombs.

BIBLIOGRAPHY

Reorganization of Mathematics in Secondary Education, a report of the National Committee on Mathematical Requirements (Houghton Mifflin, 1927), pp. 31–32, 48–50.

Teaching Junior High School Mathematics, H. C. Barber (Houghton Mifflin) pp. 48–74.

Fifth Yearbook, Teaching of Geometry, National Council of Teachers of Mathematics.

Differential and Integral Calculus, C. E. Love (1927) pp. 43–47.

The Calculus of Variations, G. A. Bliss (Open Court) pp. 7–8, 119–122, the soap film problem.

Possible Worlds, J. B. Haldane, "On Being the Right Size," Harpers.

Second Course in Algebra, Barber, pp. 248 ff., Types of Linear Dependence.

Technics and Civilization, Lewis Mumford (Harcourt Brace, 1934).

Science of Life, Wells, Huxley, and Wells (Doubleday Doran, 1931) Vol. II, pp. 738–773; Vol. III Animal Life in the Sea; Vol. IV Insect Life.

Field Book of Insects, Frank E. Lutz (G. P. Putnam's Sons).

Butterfly Book, Holland.

Insect Book, L. O. Howard.

Guide to the Study of Fresh Water Biology, American Viewpoint Society (1927), especially p. 86, Study 22: The efficiency of the streamlined form, an experiment.

Teacher Solidarity

TWENTY-FIVE years ago, I belonged to the bricklayers' union. I think my dues amounted to about twenty-five dollars a year. It would be well for teachers who seem to find it difficult to pay the modest dues of their professional organizations to remember that workers in other fields have discovered a relationship between strong, well-financed organizations and the average income of the individual members who compose them. When reverses are sustained, when school terms are shortened, curriculums trimmed, and salaries cut, is precisely the time to strengthen co-operative action to improve conditions.—J. W. STUDEBAKER, U. S. Commissioner of Education, in *The Tennessee Teacher*.

THE ART OF TEACHING

An Attempt to Develop Number Sense

By A. C. Nelson
Country Day School, Grosse Pointe, Michigan

Teacher: "What is the distance from the Equator to the North Pole?"

Pupil: "I don't know."

Teacher: "Well, about how far would you guess it is?"

Pupil: "Five miles."

The above incident occurred in a Seventh Grade arithmetic class; while this may be considered an extreme case, yet it is astounding how little number sense has been developed in pupils after seven or eight years of work in mathematics.

From the very first grade determined effort should be exerted to place as much arithmetic as possible on the understanding level of the child. Whenever possible, it should be made a part of his own experience, and while this may seem like a waste of time to some teachers, I am sure that it will pay good dividends eventually. Have the child see an inch, a foot, a yard and other common measures. Teach him to compare, to estimate, and to express in his own words the mathematical relationships in his work.

As the child's background grows, a wider field of activities is presented. Junior High School mathematics affords excellent opportunities for develping number consciousness. One worth while practice is to estimate lengths, widths, and capacities, and then actually measure them.

The functional concept offers the chance to study variable or changing quantities. Everyone realizes how often in life he encounters variation, or dependence of one quantity upon another. Consequently, it is to the student's advantage to meet this important concept early in his mathematical career and as early as the Seventh Grade it can be introduced, and enlarged in scope with each succeeding year. Questions for consideration may vary from simple ones like: "What are some of the things that effect the growth of a flower?" to more difficult ones like: "If the radius of a circle is doubled, what effect does it have on the area?" An infinite number of questions may be formulated, and pupils will welcome the chance to make similar questions once they understand what is wanted. Furthermore, pupils like to discuss these problems, and this expression of ideas should be welcomed.

Another interesting procedure designed to increase the meaning of numbers, may be termed "Current Mathematical Events." The pupils bring into the classroom published articles involving numbers, and then they interpret these or restate them in other ways. For example, a boy finds that an airplane recently traveled 300 miles per hour. This can be expressed in feet per second. The time required, at this rate, to span the continent may be computed. This rate may be compared to that of a fast express train, or an automobile's speed. Similarly financial and business reports of many kinds can be discussed in class and problems worked from them.

Careless reading is, no doubt, a serious handicap in problem analysis. One interesting aid to encourage careful reading is to request pupils to clip advertisements which require scrutinizing before their correct significance is understood. Many advertisements convey quite a different meaning when analysed than they do when only casually observed. For example: "Use X gasoline, it is 20% better."

This is meant to create the impression of superior quality, but does it actually say so? Is it 20% better than kerosene, or 20% better than water? It seems safe to suggest that millions of dollars are spent on articles which are considerably different from that which the purchaser first thought them to be.

In teaching mathematics we should attempt to carry the meaning of numbers to the pupil so that ordinary matters of quantity may be understood and visualized. We should strive for meaningful work versus vague manipulation of numbers; we should seek an understanding basis for problems rather than a memorization of rules; we should try to teach the student to visualize the action occurring in a problem, rather than the attempt to select the right process for solution by blind choice.

If a teacher of mathematics is constantly alert for opportunities and ways of improving a child's number sense, the teaching of mathematics becomes a much more interesting and fascinating subject and pupils will enjoy their arithmetic classes to a greater extent than previously, and the student will benefit substantially.

Skeleton Plan for The National Council of Teachers of Mathematics Program at San Francisco, July 1939

N. E. A. Theme: *Civic Responsibility*

N. C. T. M. Theme: *Teaching Mathematics to Meet Social Needs*

Monday Afternoon—July 3

I. Joint meeting with the Secondary Education Department.

1. E. R. Hedrick, The Contribution of Mathematics to General Education.
2. Dr. H. Geneva Lewis, Chemistry and Mathematics.

II. Arithmetic in the Modern School.

1. Paul Hanna, The Demand which Modern Elementary Curriculum Makes on Arithmetic.
2. C. C. Trillingham,
3. Mrs. Alta Harris, Discussion Leader.

Tuesday Afternoon—July 4

III. Arithmetic Progressively Taught.

1. Mrs. Lorraine Sherer, Opportunities for Arithmetic in the Activities Program

IV. High School Mathematics Which Meet Social Needs.

1. Earl Murray
2. H. M. Bacon
3. Mrs. Bruce Sumner, Mathematical Possibilities in Environmental Needs.

Wednesday Afternoon—July 5

V. Discussion Luncheon or Possibly an Evening Dinner.

VI. Junior High School Mathematics Which Functions.

1. Sophia Levi.

VII. Junior College and Teacher Training.

1. G. C. Evans.
2. F. L. Griffin.

EDITORIALS

New Syllabus in Mathematics for New York City Junior High Schools

THE New York City junior high schools are going to break away from the traditional method of teaching junior high school mathematics. Beginning at once the new mathematics will be tried experimentally in the 7A grades in twenty-five junior high schools. It will be extended to the entire system in the Autumn. According to the New York *Times* of Saturday February 25th

> Instead of using vague symbols and "textbook" illustrations, the students in the early stages will get examples taken from every-day life.
>
> "We hope that as a result of the new syllabus in mathematics failures will be decreased," Dr. Loretto M. Rochester, assistant superintendent in charge of the junior high school division said.
>
> The syllabus provides that each student be treated as an individual, and the work adjusted to meet his needs and geared to his level. The problems in the modernized course, it is said, should come within the experience and interest of the pupil.
>
> At the present stage of experimentation, the informal geometry in the seventh, eighth and ninth grades will not include formal proofs. Much of the geometry in this period is to be "essentially the geometry of everyday life."
>
> Teachers are warned to avoid excessive use of the blackboard, and to allow pupils to make their own diagrams and models. Geometric forms can be found in the pictures and models of the early shelters of primitive peoples, the report points out, referring to such subjects as the tepee, wigwam, igloo, lean-to, and square house.
>
> The objectives of the new course, it was said, are to develop ability to interpret and master the simple mathematical situations met by the average person in home or at business, and to understand the importance of graphic representation in everyday life. It is also hoped that the pupils will develop independence of thought and the habit of verifying conclusions.

This decision to try out this new program in mathematics is in line with the recent recommendations of the Joint Commission of the Mathematical Association of America and The National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education." This experiment will be watched with interest by teachers everywhere and ought to be helpful in settling some of the most important of our curriculum questions. The New York State Program in Arithmetic recently published has a great deal of the flavor of this new experiment in New York City all of which is a sign that progress is being made.

W.D.R.

The Status of Algebra

EVIDENCE is beginning to accumulate that certain communities throughout the country that discarded ninth grade algebra from the course of study are putting the subject back where it used to be. It is to be expected that when parents realize what their children are losing by having no training in mathematics, they will demand a hearing. This, however, makes it all the more important for teachers of mathematics to see to it that the kind of algebra that is taught is of the twentieth century variety and not of the seventeenth and eighteenth century as is so often the case in too many schools.

Not all of the schools will adopt the general mathematics program for some-time, but they should see to it that some improvement be made in the traditional offering.

W.D.R.

IN OTHER PERIODICALS

By NATHAN LAZAR

Alexander Hamilton High School, Brooklyn, New York

1. Benz, Carl A., "The Use of Road Maps in Elementary Algebra." *School Science and Mathematics*, 39: 2. January, 1939.

In teaching graphing it is quite helpful to use a road map to supplement the usual method of first teaching for graphs. Road maps also have value in that by their use one can teach the students how to find a point when the abscissa and ordinate are given, and also to find the abscissa and ordinate when a point is given. Other utilization of road maps are also pointed out.

2. Christofferson, H. C. "Some Mathematical Concepts to Be Taught in the Tenth Grade, or Teaching Geometry As a Way of Thinking." *School Science and Mathematics*, 39: 29–38. January, 1939.

The purpose of the paper is threefold: (1) To review some of the professed claims for geometry and geometry teaching; (2) By analysis of a simple geometry exercise to point out the salient characteristic of the rigorous reasoning which geometry illustrates; (3) To apply the same fundamental ideas about clear thinking and sound reasoning to a few typical non-geometric situations.

In the first section eight authorities are quoted to lend support to the thesis that mathematics teachers should teach more than geometry in their geometry classes.

In the second section the well known theorem is demonstrated that the north latitude of a point in the northern hemisphere is equal to the elevation of Polaris; this is done in order to illustrate the fact that the conclusion is dependent upon certain definitions and certain previously accepted statements called axioms, postulates, and theorems, and that the conclusion is true only as these previously accepted and established statements are true.

The third section is replete with interesting material and techniques showing how the patterns of reasoning in geometry may be carried over to the reasoning with non-geometric situations. It is particularly heartening to read the following comments on indirect proofs and converses: "Converses and their close friend, indirect proof, form a problem which alone merits the attention of a paper. . . . They are rather inadequately treated in most geometries. Converses are usually not true, yet few if any are mentioned in geometry except those that are true."

3. Dockeray, N. R. C. "The Law of Quadratic Reciprocity." *The Mathematical Gazette*, 22: 440–453. December, 1938.

The author apologizes for reproducing in a magazine selected passages from standard mathematical works, on the ground that he hopes that they may prove useful to those who have neither the leisure nor the opportunity to become acquainted with the subject from the pages of Mathews or Dickson.

After a lucid explanation of the term "Quadratic Residues" the author proceeds to a detailed proof of the law. The article closes with the application of quadratic residues to the factorization of large numbers, and to the proof that there is an unlimited number of primes of the form $4n+1$.

4. Schaaf, William L., "The Education of Mathematics Teachers." *National Mathematics Magazine*, 13: 83–89. November, 1938.

An interesting article on an important topic. The writer is inclined to regard "the candidate's entire academic achievement (as expressed in credits, grades, courses, etc.) in pure mathematics, cultural background, and educational theory alike merely as an *initial criterion* in the ultimate selection of the teacher. The eventual solution of the problem of selecting teachers who know what both to teach and are themselves educated, lies with schools and colleges that prepare them and not primarily with accrediting agencies, extra-mural examinations, standards of certification, degrees, or scholastic records."

The writer offers in conclusion several possible avenues for effecting the desired integration.

5. Shaw, Allen A., "A Pre-Euclidean Fragment of the *Elements*." *National Mathematics Magazine*, 13: 76–82. November, 1938.

The author describes the contents of an Armenian manuscript the original of which is at the University of Pavia, Italy, and has neither date, nor name of translator. The following are the conclusions he arrives at:

(a) "The Armenian manuscript is a frag-

ment of pre-euclidean *Elements* written by Leon of Theudius of Magnesia whose works on the *Elements* are lost.

(b) "The Armenian manuscript is the *first* specimen of pre-euclidean *Elements*, preserved through the medium of Armenian translation.

"With regard to date of translation of the Armenian fragment, close study of style and diction leads the present writer to believe that the translation may be as early as the fifth century."

A translation of the fragment as well as a photostatic reproduction of it are included.

6. Thomson, James, "On Inaccuracies in Style Which Frequently Occur in Mathematical Composition." *The Mathematical Gazette*, 22: 426–428. December, 1938.

An interesting quotation from Thomson's *Elementary Treatise on Algebra*, 6th Edition, 1850.

After enumerating many errors in expression that occur in daily speech, in writing as well as in textbooks, the writer concludes that "many other instances of careless and incorrect modes of expression might be adduced. Those, however, that have been adverted to above will turn the attention of the intelligent student to the subject, and will make him feel that while accuracy in reasoning is the thing mainly to be attended in his investigations, yet correctness and elegance of expression in the communication of his ideas are matters of much importance. He should recollect also, that, while no person has ever acquired a faultless style, as little as anyone has succeeded in writing even moderately well without studying with care and attention the means to be pursued for accomplishing the object."

To all of the above remarks, the editors of mathematical journals, their readers, and compositors will fervently say "Amen."

7. Wood, F. J., "Impressions of School Mathematics in the United States." *The Mathematical Gazette*, 22: 461–465. December, 1938.

The writer spent a year in the United States as visiting instructor of mathematics in a large preparatory school, a boarding school of 750 boys with a staff of about eighty-five men, twelve of whom form the mathematics department. (The name of the school is not given.)

"It is evident from the various American publications on the subject that thoughts of coordinating the various branches of mathematics have been in the minds of American educationists for many years. Yet a most unsatisfactory breach between algebra, geometry, and trigonometry still exists. . . . Most of the school textbooks in mathematics—particularly those in geometry—still show traces of the influence of Wentworth. . . . There is, in general, little or no mention of converses of the main proposition on angles in a circle, so that little is done on concyclic points. . . . In solid geometry, the course has been much more detailed and rigorous than we are used to, though it is now in the process of being cut down. . . . The separation of mathematics and physics is rather disconcerting to a teacher from England for owing to the choice that a boy can make in his courses, his advance in physics is not necessarily correlated with his mathematics, and in consequence any appeal to physical facts is likely to fail of its effect. But things are moving in this connection, though the course system which is firmly established will still prove an obstacle. . . . Arithmetic as such finds no place in the curriculum. The expectation—somewhat optimistic—is that boys will know all about it. . . . It should be said in passing that while an American B.A.'s knowledge in his major subject will not be as extensive as it would be in [England], he will often stay on at his university to specialize for a master's degree, and though he will then be older than the English B.A. he will have a knowledge of and an interest in other subjects which some of our over-specialized graduates sadly need."

8. Yates, Robert C., "Line Motion and Trisection." *National Mathematics Magazine*, 13: 63–66. November, 1938.

A description of the mathematical theory and construction underlying a linkage that will trisect any angle. A short bibliography and three diagrams are included.

Notice to Readers of The Mathematics Teacher

Please do not send money, stamps, or checks to *The Mathematics Teacher* in payment for any of the National Council Yearbooks or *Numbers and Numerals*. All such payments should be sent to *The Bureau of Publications*, Teachers College, Columbia University, 525 West 120th Street, New York, N.Y.

NEWS NOTES

The Men's Mathematics Club of Chicago and the Metropolitan Area discussed the teaching of mathematics in junior college at their January dinner meeting. Talks were given on the following topics: Reorganization of the First Year College Mathematics and a Testing Program, The Survey Course in Junior College Mathematics, Why Introduction of Calculus in First Year, A New Course in Business Mathematics, and Visual Aids to Instruction in Mathematics.

The Women's and Men's Mathematics Clubs of Chicago have maintained a permanent exhibit of interesting models and other mathematical developments from the class rooms for the past year and a half. This exhibit has been housed in a booth on the first floor of the Planetarium with the co-operation of Miss Maude Bennot, Director of the Planetarium.

Exhibits have been shown from the following schools: Lane Technical High School and Oak Park High School June 12–October 10, 1937; Evanston High School and Tilden Technical High School, October 10, 1937–January 2, 1938; Foreman High School and Hyde Park High School, January–April, 1938; and Dundee Community High School and Saint Xavier College, April–July, 1938.

During the summer, the clubs showed selected material from all the work of the year. This fall work in Mathematics from the Gary Public Schools and from J. Sterling Morton Junior College has been shown. The winter exhibit consists of work from Pullman Free School of Manual Training, from Christian Fenger High School and from J. Sterling Morton Junior College.

The annual joint banquet of the Men's and Women's Mathematics Clubs of Chicago and the Vicinity took place February 4, at six o'clock, in the Chicago Woman's Club. Mrs. Frances Mullen was chairman of arrangements.

Professor Arthur Haas, of the University of Notre Dame, addressed the combined clubs on "The Modern Conception of the Physical World." Professor Haas is especially known for his researches in atomic physics, quantum theory, relativity, spectroscopy, and astrophysics. At present, he teaches theoretical physics, atomic physics, and wave mechanics at Notre Dame.

The Oklahoma Section of the Mathematical Association of America held special meetings during the convention of the Oklahoma Educational Association in Tulsa, February 9 through 11.

The following addresses were given: "Remarks on the Teaching of College Geometry," by J. H. Butchart, of Enid; "The History of the Weierstrass Non-differentiable Function," by W. C. Randels, of Norman; "The General Equation of the Second Degree in Plane Analytical Geometry," by A. H. Diamond, of Stillwater; "Special Tetrahedrons," by N. A. Court, of Norman; "The Place of Mathematics in General Education," by Raleigh Schorling, Ann Arbor, Mich.

Professor Shorling was the principal speaker at the meeting of the Oklahoma Mathematics Council, taking for his subject "Remedial Instruction at the Secondary School Level."

Teachers College, Columbia University, will offer the following courses in the teaching of mathematics this summer, July 5 through August 11. By Professor W. D. Reeve: Teaching Algebra in Secondary Schools, and The Reorganization of Secondary School Mathematics; Professor John R. Clark, Teaching Intuitive Geometry in Junior High Schools, and Teaching Geometry in Secondary Schools; Professor Carl N. Shuster, Modern Business Arithmetic, Methods of Teaching in Junior and Senior High Schools, and Field Work in Mathematics; Dr. John Swenson, Professionalized Subject Matter in Senior High School Mathematics, First Part, and Professionalized Subject Matter in Senior High School Mathematics, Second Part: Miss Ethel Sutherland, Teaching Arithmetic in Primary Grades, First Three Grades, Teaching Arithmetic in Intermediate Grades, Fourth, Fifth, and Sixth Grades, and Professionalized Subject Matter in Junior High School Mathematics; Dr. Nathan Lazar, Teaching Algebra in Junior High Schools, and Experimental Demonstration Class in Demonstrative Geometry. Mr. Max Black, Teaching Mathematics in the Secondary Schools of England, and An Introduction to Mathematical Methods; Dr. Joseph Lauwerys, The Correlation of Mathematics and Science in Secondary Schools.

Problems of Plateau and Dirichlet and more general multiple integral problems of the calculus of variations will be stressed in the Mathematics Department conference and seminar at the University of Chicago summer session, June 27 through 30.

This conference and seminar is one of twelve special institutes which have been arranged for the summer session, supported by courses planned to supplement the fields of the various special conferences.

Additional emphasis will be placed in the Education Department at Chicago this summer on the opportunity for students in education to earn the master's degree without writing a thesis. This plan, inaugurated last year by the department, provides an alternative to the traditional thesis requirements, permitting teachers to put the emphasis of their study on a broad course of training.

The British Mathematical Association held its Annual Meeting at King's College, London, on January 2 and 3. Some of the important addresses and papers given at this meeting are as follows:

The Presidential Address, by W. Hope-Jones "Simplicity and Truthfulness in Arithmetic"; by Professor W. L. Bragg, "The Symmetry of Patterns"; by A. Buxton, "The Teaching of Applied Mathematics in Technical Colleges"; by H. W. Newton, "Greenwich Observatory: Some Aspects of Its Work"; by W. L. Ferrar, "Algebra in the Higher School Certificate"; by Lord Stamp, "Education and the Statistical Method in Business (with Special Reference to Railway Statistics).

Edwin A. Beito, Program Chairman of the Wichita Mathematics Association, Wichita, Kansas, has announced the special speakers and subjects for spring meetings of the Association.

At a dinner meeting, March 23, Dr. Allan R. Congdon, of the University of Nebraska, will speak on "Handicaps of Adults Due to Insufficient Training in Mathematics."

The April meeting will be devoted to a special program and the election of officers.

During the past year worth while programs have included Dr. U. G. Mitchell, of the University of Kansas, speaking on "The Preliminary Report of the National Committee"; and Professor Reagan, of Friends University, whose subject was "Mathematical Recreations."

NEW BOOKS

A Course in General Mathematics. Revised edition. By Clinton Harvey Currier, Emery Ernest Watson, and James Sutherland Frame. The Macmillan Company, 1939. 382 pp. Price, $3.00.

Teachers and students familiar with the 1929 first edition of the book by Professors Currior and Watson will be interested in finding and weighing the value of the changes and improvements in the revision. Professor Frame, colleague at Brown University of Professor Currier, has been added as a co-author on the basis of the fact that he used the original textbook for three years in his own classes at Brown. Surely this original and exceptionally intelligent reason for bringing in a co-author on the revision of a textbook should have a heartening effect on the loyalties of friends of the old book.

This text is intended for the first (and probably the last) year of college mathematics. Its prerequisites are elementary algebra and plane geometry. On this slight foundation, elements of algebra, trigonometry, analytic geometry, and calculus are taught, the topics chosen to illustrate principles and the algebra as simple as possible in order to direct attention to the method. There is no pretense at completeness or the development of a high degree of skill.

Within the volume are tables of powers and roots, four place logarithms, four place trigonometric functions, compound discount, annuity tables, a table of exponential and hyperbolic functions, answers to exercises, and a four page index.

The attractive green binding and reasonable size, $5\frac{1}{2}$ by $8\frac{1}{4}$, make it a somewhat more friendly book than the average college mathematics text.

E.W.

The Psychology of Secondary-School Teaching. Revised and enlarged edition. By James L. Mursell. W. W. Norton and Company, Inc., 1939. Price, $2.75.

Since Professor Mursell, of Teachers College, Columbia, brought out the first edition of his book in 1932, important research and further contributions to educational psychology have been made. These have been of such consequence as to necessitate this revised and enlarged edition.

Here is a psychology textbook written with the definite and practical purpose of helping teachers improve instruction and control the conduct of their students in secondary school. There are chapters aimed to show administrators how a thorough and up-to-date grounding in psychological theory and methods may help in adminstration and in the guidance of pupils. Both administration and teachers may concern themselves with the relation of modern educational psychology with setting standards, as described by Professor Mursell.

There are no chapters on the teaching of special subjects. Rather five types of mental ability are defined and discussed, and reference is made through them to the special subjects. The five types of mental ability required in most academic work are, according to the author, (1) reading, (2) English use, (3) attitude and appreciation, (4) problematic thinking, and (5) memory. This avenue of approach to school learning has evidently been of interest and value to many teachers, since the success of the first edition has made the labor of the present revision seem worth while to the author and his publishers.

E.W.

The College Journey. An Introduction to the Fields of College Study. Edited by Ronald B. Levinson. Thomas Nelson and Sons, 1938. 569 pp. Price, $2.50.

This book, designed to be used as a general orientation text for college freshmen, might well be on the shelves of the high school library. The junior or senior looking toward college will find in it answers to many questions that are or should be in his mind.

Many college and universities have tried to show entering students something of the richness of what lies before them in the years of academic learning. So-called orientation courses, reading seminars, and lectures have been used to acquaint the students with different fields, to remove prejudices and bridge the gaps of ignorance that prevent the intelligent choice of courses throughout their college careers. In the book *The College Journey*, Professor of Philosophy Levinson and his colleagues at the University of Maine have worked out an imposing sight-seeing itinerary through the mysterious land that lies ahead of the college freshman.

The book is divided into two parts. Part I, "The Development of the Arts and Sciences,"

presents the historical evolution of each of the subjects treated. These subjects are grouped as (1) Mathematics and the Natural Sciences, including besides mathematics, astronomy, physics, chemistry, geology, and biology; (2) The Social Sciences, including psychology, education sociology, economics, government, and history; (3) The Humanities, including languages, literature, music, fine arts, religion and philosophy.

As stated in the preface "In Part II the reference to history and development is abandoned in favor of an emphasis upon the present state of affairs within each particular field of study. Our effort here is to bring before the student, in miniature, a visualization of the *scope*, *affiliations*, and major *applications* of each field, and to point a prophetic index finger down the major highways that lead beyond the campus into what is called 'Life,' to exhibit the continuity of the Arts and Sciences with the art and business of living in the contemporary world. In so doing it has been necessary to paint in broad strokes and often with an impressionistic technique."

Teachers of mathematics will be pleased with the historical presentation of their field by Professor Levinson and with the sketch of its contemporary content and status by Warren S. Lucas and Spofford H. Kimball. The pleasant, almost humorous tone of these articles has in it much of salesmanship for the subject. At the same time, mathematics teachers should be tempted by similar treatment of other fields in the social sciences and humanities to do some pleasurable, even substantial reading to the refreshment of mind and spirit.

Books that claim to give in short sketches the high points of the several academic fields of knowledge have a great appeal to the curious student or layman. They seem to promise so much, a comprehension of all important knowledges in ten easy lessons. Such books are often laid down, however, with a feeling of having been beguiled. The articles may be mere summaries of important facts, too briefly stated without the developmental material necessary to build up understanding.

Such a unhappy condition does not follow reading this book. The authors have purposely avoided trying to produce "an encyclopaedia on the back of a penny." They have selected material and shaped their discussion in each field to demonstrate the essential character of the subject rather than its essential facts. Moreover, the organization and content of the chapters are arranged to show the continuity and interrelation of human knowledges and to confute the fallacy of the separation of the cultural and the practical.

E.W.

Mathematical Adventures. By Fletcher Durell. Boston, 1938. Bruce Humphries, Inc., 157 pp. Price, $2.00.

Dr. Durell has assembled professional writings of the past dozen years in this volume. Six of its twelve chapters have appeared as articles in School Science and Mathematics between January, 1927, and December, 1931. Four were published in The Mathematics Teacher between November, 1928, and November, 1932. The last two chapters were given as lectures before the Mathematical Club of Temple University and have now been published for the first time.

"Co-operative Mathematics," title of the first article, states the important theme of the series, that while each department of mathematics should be taught "as a distinct discipline," nevertheless, material should be transferred from other departments to the one being studied when the borrowed materials illustrate the principles involved in the work under discussion. Not for Dr. Durell, Head of the Mathematics Department at Lawrenceville School, is the "integrated mathematics" of Swenson and others for whom "subject matter barriers" should be let down. Rather let traditional methods be modified but let's stay away from a fused mathematics.

Dr. Durell is in camp with a number of important leaders in his own field and in education, but his viewpoint hardly fulfills the promise of the volume's title *Mathematical Adventures*, unless it be an adventure for an "essentialist" to raise his head and restate the arguments of old time, that students should study arithmetic, algebra, geometry, and other departments of mathematics as such.

Another chapter, "Three States of Mastery," contributes a valuable idea to the problem of the psychological hurts sometimes inflicted upon pupils by the segregation of classes according to their mathematical abilities. Dr. Durell is in favor of the segregation, but he has rather charmingly dispelled the dark cloud that hangs over the sections of lower abilities by pointing out that relatively low mathematical ability need not mean a low natural ability in academic subjects but may be the result of poor or insufficient mathematical instruction in earlier years. Even a very bright pupil whose interests are not in mathematics may wish to be in the lower class in order to be able to give as little time as possible to mathematics and so be set free to develop his more promising aptitudes.

Although the material on "Three Stays of Mastery" was published in The Mathematics Teacher, November, 1938, under the title "Ability Grouping in Mathematical Classes," it is still important, and the reviewer is pleased

to see it emphasized by inclusion in this bound volume.

The last two chapters, hitherto unpublished, are on mathematical recreations and the fourth dimension. The latter is by no means completely taught, but it is facilely illustrated by analogous situations and anecdotes in the lives of the people of Thinland.

E.W.

Theorie und Praxis der geometrischen Konstruktionsaufgaben. By Dr. W. Lietzmann. H. L. Schlapp, Darmstadt, 1935. pp. 28. Price 1 R.M.

Geometrische Konstruktionen bei beschranktem Gebrauch des Zirkels. By Dr. H. Fuhr. Emil Roth, Giessen, 1930 pp. 18. Price 1.60 R.M.

Geometricheskije postrojenia i priblizhenija (Geometrical constructions and approximations). By Professor N. F. Tschetverukhin. State Pedagogical Publications, Moscow, 1935. pp. 80. Price 80 kopecks (about 16¢).

The problem of geometrical constructions seems to be more popular in Europe than in this country. We cannot even boast of a single book on this topic ever published here (unless we mention a monograph by B. Alvord on Apollonius circles published by the Smithsonian Institution in 1855). We have an American edition of Peterson and a British publication by H. P. Hudson ("Ruler and Compasses"). But if one is to try to get anywhere in the field of constructions he necessarily is compelled to use foreign sources. And even these are scattered either in magazines and journals or in treatises written in a language so highly technical that an average teacher may as well abandon hope before he attempts to read them.

The three pamphlets are quite recent publications. They are designed to meet the needs of teachers; they are not "popular editions." But the exposition in them is rather simple.

Dr. Lietzmann's aim is to give a bird's eye view of the field of geometric constructions, and he has successfully crowded into twenty eight small pages discussions of (a) Euclidean constructions, (b) limitations that might be imposed, (c) some extensions of construction tools, (d) theoretical bases of construction problems, (e) practical performance of constructions. His bibliography at the end is suggestive and very useful. The book bristles with many ideas. The illustrations are interesting. It is not a textbook, however, but it gives one a feeling of how much there is in this field *and how much is still left to be done.* The notions of geometrography, constructions in a limited plane, approximations, constructions by right angles, by compasses alone, by other tools (in the realm of the Euclidean sense, that is, the Platonic restriction), these are but a few topics that Dr. Lietzmann succinctly presents, not with the view that a teacher should become the master of them, but with the aim to stimulate him to further study.

Dr. Furst's aim is to survey the so-called Steiner constructions, that is constructions with the aid of a fixed circle. Prior to the introduction of Steiner's constructions he discusses constructions related to the fixed circle method. There is some genetic reason for this in Dr. Furst's procedure. He thus builds up the entire field of the so-called "limited constructions." There isn't anything new in his presentation. This pamphlet is suggestive of a survey. It represents a report of the padagogical seminar held at the Oberrealschule in Giessen. But there is enough material in this monograph to keep a teacher busy and above all there are sufficient suggestions in the eighteen pages to enable a resourceful teacher to make good use of it in his geometry classroom.

The Russian monograph reviewed is an attempt (and a very successful one) to make readable and really digestible what Th. Vahlen presented in his classical work "Konstruktionen und Approximationen." This monograph, as mentioned on its title page, is published "for pedagogical institutes and teachers of secondary schools." It also aims to fill the gap between the so-called elementary constructions taught in professionalized subject matter courses in geometry and the more advanced problems in geometrical constructions. It begins with a general discussion of ruler and compasses constructions. It begins with a general discussion of ruler and compasses constructions which is treated algebraically. Then the problem of Castillon and its generalization is fully presented (those who are weak in Russian might consult Vahlen's book, section 52). Then the problem of Apollonius and its generalization is presented. Cubic and fourth degree approximations, the principle of tangents, the problem of Pappus, precision of the approximations, these are some of the topics presented. The three problems of antiquity were not overlooked, and some of the angle trisectors, circle squarers would find in those pages the shock of their lifetime. The author made use of materials published by him in various journals as well as of materials published by others since Vahlen's book was published (1911). In this respect some of the phases of geometrical approximations are brought up to date.

Aaron Bakst